VIBRATION ANALYSIS OF COMPOSITE BEAM WITH CRACK

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

Master of Technology

In

Structural Engineering

By

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ROURKELA-769008

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CERTIFICATE

This is to certify that the thesis entitled, ***"VIBRATION ANALYSIS OF COMPOSITE BEAM WITH CRACK"*** *submitted by* ***Mr. Abdul Khader Shahadaf TH*** *in partial fulfillment of the requirements for the award of Master of Technology Degree in Civil Engineering with specialization in "Structural Engineering" at National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance. To the best of my knowledge, the matter embodied in this thesis has not been submitted to any other university/ institute for award of any Degree or Diploma.*

Prof. Shishir Kumar Sahu

Dept. of Civil Engineering,

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ABSTRACT

Composite beams and beam like elements are principal constituents of many structures and used widely in high speed machinery, aircraft and light weight structures. Crack is a damage that often occurs on members of structures and may cause serious failure of the structures. The influence of cracks on dynamic characteristics like natural frequencies, modes of vibration of structures has been the subject of many investigations. However, the parametric studies like effect of geometry, crack location and support conditions on natural frequencies of composite beam are scarce in literature. In the present work, a numerical study using finite element is performed to investigate the free vibration response of composite beams. The finite element software ANSYS is used to simulate the free vibrations. A variety of parametric studies are carried out to see the effects of various changes in the laminate parameters on the natural frequencies. The parameters investigated include the effects of fiber orientation, the location of cracks relative to the restricted end, depth of cracks, volume fraction of fibers, length of beam and support conditions. The study shows that the highest difference in frequencies occur when the value of the fiber orientation equal to zero degree. The increase of the beam length results in a decrease in the natural frequencies of the composite beam and also shows that an increase of the depth of the cracks leads to a decrease in the values of natural frequencies.

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NOMENCLATURE

Notation	Description
F	Cross-sectional area of the element
a/H	Crack depth
α	Angle of the fiber
B	Width of the composite beam
H	Height of the composite beam
I	Moment of inertia
L	Length of the composite beam
L_1	Crack location
q	Vector
ρ	Mass Density of the beam
ω	Natural frequency
$\boldsymbol{\omega_n}$	Non-dimensional natural frequency
[K]	Stiffness matrix
[M]	Mass matrix

Chapter 1

INTRODUCTION

1.1 Introduction

In the recent decades, fiber reinforced composite materials are being used more frequently in many different engineering fields. The automobile, aerospace, naval, and civil industries all use composite materials in some way. Composite materials are gaining popularity because of high strength, low weight, resistance to corrosion, impact resistance, and high fatigue strength. Other advantages include ease of fabrication, flexibility in design, and variable material properties to meet almost any application.

The airplane F.111 was one of the first models to incorporate this technology. As another example, the airplane Boeing 767 has 2 tons in composite materials (Tsai, 1986). The possibility to combine high strength and stiffness with low weight has also got the attention of the automobile industry: the Ford Motor Company developed in 1979 a car with some components made from composite materials. The prototype was simply 570 kg lighter than the version in steel, only the transmission shaft had a reduction of 57% of its original weight (Dharam, 1979). Besides these examples in the aeronautical and automobile industry, the application of composite materials have been enlarged, including now areas as the nautical industry, sporting goods, civil and aerospace construction. On the other hand the use of laminated composite materials as a coating for isotropic materials may well prove valuable for structural purposes such as the reduction of vibrations and noise in sheet metal frames of machines and an increase in the wear resistance at contact surfaces.

Beams and beam like elements are principal constituents of many mechanical structures and used widely in high speed machinery, aircraft and light weight structures. Fiber-reinforced laminated beams constitute the major category of structural members, which are widely used as movable elements, such as robot arms, rotating machine parts, and helicopter and turbine blades. Similar to other structural components, beams are subjected to dynamic excitations. Reducing the vibration of such structures is a basic requirement of engineers. One method to reduce the vibration of a structure is to move its natural frequencies away from frequency of excitation force. There are different methods to modify the natural frequencies of beam structures.

In general, any continuous structure has infinite degrees of freedom and, consequently, an infinite number of natural frequencies and the corresponding modal shapes. If a structure vibrates with a frequency equal to a natural one, the vibration amplitude grows rapidly with time, requiring a very low input energy. As a result, the structure either fails by overstressing, or the nonlinear effects limit the amplitude to a large value, leading to high-cycle fatigue damage. Thus, for any structure, its natural frequencies must be determined in order to ensure that the loading frequencies imposed and the natural frequencies differ considerably; in other words, to avoid resonances.

To avoid structural damages caused by undesirable vibrations, it is important to determine:

1 - Natural frequencies of the structure to avoid resonance;

2 - Mode shapes to reinforce the most flexible points or to determine the right positions to reduce weight or to increase damping;

3 - Damping factors.

With respect to these dynamic aspects, the composite materials represent an excellent possibility to design components with requirements of dynamic behavior (Tita, 2003).

During operation, all structures are subjected to degenerative effects that may cause initiation of structural defects such as cracks which, as time progresses, lead to the catastrophic failure or breakdown of the structure. Thus, the importance of inspection in the quality assurance of manufactured products is well understood. Cracks or other defects in a structural element influence its dynamical behavior and change its stiffness and damping properties. Consequently, the natural frequencies of the structure contain information about the location and dimensions of the damage (Krawczuk, 1995).

Structural damage detection has gained increasing attention from the scientific community since unpredicted major hazards, most with human losses, have been reported. Aircraft crashes and the catastrophic bridge failures are some examples. Development of an early damage detection method for structural failure is one of the most important keys in maintaining the integrity and safety of structures. The cracks can be present in structures due to their limited fatigue strengths or due to the manufacturing processes. These cracks open for a part of the cycle and close when the vibration reverses its direction. These cracks will grow over time, as the load reversals continue, and may reach a point where they pose a threat to the integrity of the structure. As a result, all such structures must be carefully maintained and more generally, SHM denotes a reliable system with the ability to detect and interpret adverse “change” in a structure due to damage or normal operation (Ramanamurty 2008).

The greatest challenge in designing a SHM system is to identify the underline changes due to damage or defect. Lots of damage detection techniques have been proposed for structural health monitoring. Some of the nondestructive evaluation approaches that utilize technologies such as X-ray imaging, ultrasonic scans, infrared thermograph, and eddy current can identify damages. However, they are somehow difficult to implement, and some of them are impractical in many cases such as in service aircraft testing and in-site space structures. Almost all of the above techniques require that the vicinity of the damage is known in advance and the portion of the structure being inspected is readily accessible. The drawbacks of current inspection techniques have led engineers to investigate new methods for continuous monitoring and global condition assessment of structures. That is the case for methods based on vibration responses that allow one to obtain meaningful time and/or frequency domain data and calculate changes in the structural and modal properties, such as resonance frequencies, modal damping and mode shapes, and use them with the objective of developing reliable techniques to detect, locate and quantify damage. Hence, the vibration-based damage identification method as a global damage identification technique is developed to overcome these difficulties (Ramanamurthy 2008).

Among many SHM techniques, the dynamic response-based damage detection method attracts most attention due to its simplicity for implementation. This technique makes use of the dynamic response of structures which offers unique information on the defects contained with these structures. Changes in the physical properties of the structures due to damage can alter the dynamic response, such as the natural frequency and mode shape. These parameter changes can be extracted to predict damage detection information, such as the presence, location, and severity of damage in a structure. The natural frequency provides the simplest damage detection method since damage tends to reduce the stiffness of the structure. Therefore, a reduction of natural frequency may indicate the existence of damage in the structure. However, the natural frequency is a global feature of the structure, from which the location of the damage is difficult to determine. The modal parameters (e.g., the mode shape and flexibility), which can capture the local perturbation due to damage are used in order to locate damage (Ramanamurthy 2008).

The dynamic response of structures can offer unique information on defects that may be contained within the structures. Changes in the physical properties of the structures due to damage will alter the dynamic responses such as natural frequencies, damping and mode shapes. These physical parameter changes can be extracted to estimate damage information. In the past 20 years a lot of work has been published in the area of damage detection, where various methods have been proposed. The modal parameters such as natural frequencies and mode shapes can be used to detect the initiation and development of cracks.

The fundamental idea for vibration-based damage identification is that the damage-induced changes in the physical properties (mass, damping, and stiffness) will cause detectable changes in modal properties (natural frequencies, modal damping, and mode shapes). For instance, reductions in stiffness result from the onset of cracks. Therefore, it is intuitive that damage can be identified by analyzing the changes in vibration features of the structure (Scott, 1998).

Although in vibration test, the excitation and response are always measured and recorded in the form of time history, it is usually difficult to examine the time domain data for damage identification. A more popular method is to examine the modal domain data through modal analysis technique, in which the time domain data is transformed into the frequency domain, and then the modal domain data can be further extracted from the frequency domain data. During the past three decades, great effort has been made in the researches within all three domains (i.e., time, frequency, and modal domains). It seems that this effort will continue since no single existing method can solve all the damage identification problems from various types of damages and structures. However, the modal domain methods attract more attention and play a dominant role in the state-of-the-art of structural damage identification. The modal domain methods evolve along with the rapid development of experimental modal analysis technique, and they gain their popularity because the modal properties (i.e., natural frequencies, modal damping, modal shapes, etc.) have their physical meanings and are thus easier to be interpreted or interrogated than those abstract mathematical features extracted from the time or frequency domain. During the last three decades, extensive research has been conducted in vibration-based damage identification, and significant progress has been achieved in this highlighted area. Thus, the main objective of this work is to contribute for a better understanding of the dynamic behavior of components made from fiber reinforced composite materials, specifically for the case of beams. In order to investigate the influence of the angle of fiber on the dynamic behavior of the components, numerical analysis using the Finite Element Method has been carried out (zonta 2000).

In this thesis **Chapter 2** presents the review of literature confining to the scope of the study. The vibration analysis of cracked composite beam has been briefly addressed in this chapter.

The **Chapter 3** presents methodology for the modal analysis analysis of composite beam with and without crack using finite elements method. The computer program implementation has been briefly described with the help of a flowchart. In **Chapter 4**, comparison with previous study and numerical results has been presented to validate the proposed method. The **Chapter 5** concludes the present investigation and an account of possible future scope of extension to the present study. The publications and books referred have been included in **References**.

Chapter 2

LITERATURE REVIEW

2.1 Introduction

Beams and beamlike elements are principal constituent of many structures and used widely in high speed machinery, aircraft and lightweight structures. Crack is a damage that often occurs on members of structures and may cause serious failure of the structures. The crack in a composite structure may reduce the structural stiffness and strength and consequently their static and dynamic behavior is altered. The increased use of laminated composite beams requires a better understanding of vibration and buckling characteristics of such beams. Determination of the dynamic characteristics of cracked laminated composite beams is essential not only in the design stage but also in the performance of the structure .Vibration analysis can be also used to detect structural defects such as cracks, of any structure offers an effective, inexpensive and fast means of non-destructive testing.

Cracks occurring in structural elements are responsible for local stiffness variations, which in consequence affect their dynamic characteristics. This problem has been a subject of many papers, but only a few papers have been devoted to the changes in the dynamic characteristics of composite beam. In the present investigation an attempt has been made to the reviews on composite beam with crack in the context of the present work and discussions are limited to the following area of analysis.

2.2 Review on vibration of cracked composite beam

A crack on a structural member introduces a local flexibility that is a function of crack depth. This flexibility changes the dynamic behavior of the system and its stability characteristics.

Nikpur and Dimarogonas (1988) presented the local compliance matrix for unidirectional composite materials. They have shown that the interlocking deflection modes are enhanced as a function of the degree of anisotropy in composites.

QIAN and Gu (1990) derived an element stiffness matrix of a beam with a crack from an integration of stress intensity factors, and then a finite element model of a cracked beam is established. This model is applied to a cantilever beam with an edge-crack, and the eigen frequencies are determined for different crack lengths and locations. Finally, a simple and direct method for determining the crack position, based on the relationship between the crack and the eigen couple (eigenvalue and eigenvector) of the beam, is proposed and this method can be suggested to complex structures with various cracks, if their stress intensity factors are known.

Ostachowicz and **Krawczuk** (1991) presented a method of analysis of the effect of two open cracks upon the frequencies of the natural flexural vibrations in a cantilever beam. Two types of cracks were considered: double-sided, occurring in the case of cyclic loadings, and single-sided, which in principle occur as a result of fluctuating loadings. It was also assumed that the cracks occur in the first mode of fracture: i.e., the opening mode. An algorithm and a numerical example were included.

Manivasagam and **Chandrasekaran** (1992) presented results of experimental investigations on the reduction of the fundamental frequency of layered composite materials with damage in the form of cracks.

Krawczuk (1994) formulated a new beam finite element with a single non-propagating one-edge open crack located in its mid-length for the static and dynamic analysis of cracked composite beam-like structures. The element includes two degrees of freedom at each of the three nodes: a transverse deflection and an independent rotation respectively. He presented the exemplary numerical calculations illustrating variations in the static deformations and a fundamental bending natural frequency of a composite cantilever beam caused by a single crack.

Krawczuk and **Ostachowicz** (1995) investigated Eigen frequencies of a cantilever beam made from graphite-fiber reinforced polyimide, with a transverse on-edge non-propagating open crack. Two models of the beam were presented. In the first model the crack was modeled by a massless substitute spring Castigliano's theorem. The second model was based on the finite element method. The undamaged parts of the beam were modeled by beam finite elements with three nodes and three degrees of freedom at the node. The damaged part of the beam was replaced by the cracked beam finite element with degrees of freedom identical to those of the non-cracked done. The effects of various parameters the crack location, the crack depth, the volume fraction of fibers and the fibers orientation upon the changes of the natural frequencies of the beam were studied.

Ghoneam (1995) presented the dynamic characteristics laminated composite beams (LCB) with various fiber orientations and different boundary fixations and discussed in the absence and presence of cracks. A mathematical model was developed, and experimental analysis was utilized to study the effects of different crack depths and locations, boundary conditions, and various code numbers of laminates on the dynamic characteristics of CLCB. The analysis showed good agreement between experimental and theoretical results.

Dimarogonas (1996) reported a comprehensive review of the vibration of cracked structures. This author covered a wide variety of areas that included cracked beams, coupled systems, flexible rotors, shafts, turbine rotors and blades, pipes and shells, empirical diagnoses of machinery cracks, and bars and plates with a significant collection of references.

Krawczuk, Ostachowicz and **Zak** (1997) presented a model and an algorithm for creation of the characteristic matrices of a composite beam with a single transverse fatigue crack. The element developed had been applied in analyzing the influence of the crack parameters (position and relative depth) and the material parameters (relative volume and fiber angle) on changes in the first four transverse natural frequencies of the composite beam made from unidirectional composite material.

Chondros (1998**)** developed a continuous cracked beam vibration theory for the lateral vibration of cracked Euler Bernoulli beams with single edge or double edge open cracks. The HuWashizuBarr variational formulation was used to develop the differential equation and the boundary conditions of the cracked beam as a one dimensional continuum. The displacement field about the crack was used to modify the stress and displacement field throughout the bar. The crack was modeled as a continuous flexibility using the displacement field in the vicinity of the crack found with fracture mechanics methods.

Hamada (1998) studied the variations in the Eigen-nature of cracked composite beams due to different crack depths and locations. A numerical and experimental investigation has been made. The numerical finite element technique was utilized to compute the Eigen pairs of laminated composite beams through several states of cracks. The model was based on elastic-plastic fracture mechanics techniques in order to consider the crack tip plasticity in the analysis. The model has been applied to investigate the effects of state of crack, lamina code number, boundary condition on the dynamic behavior of composite beams.

Zak, Krawczuk and **Ostachowicz** (2000) developed the work models of a finite delaminated beam element and delaminated plate element. They carried out an extensive experimental investigation to establish changes in the first three bending natural frequencies due to delamination. The subsequent results of the numerical calculations were consistent the results of the experimental investigations.

Banerjee (2001) derived exact expressions for the frequency equation and mode shapes of composite Timoshenko beams with cantilever end conditions in explicit analytical form by using symbolic computation. The effect of material coupling between the bending and torsional modes of deformation together with the effects of shear deformation and rotatory inertia is taken into account when formulating the theory. The expressions for the mode shapes were also derived in explicit form using symbolic computation.

Wang and **Inmana** (2002) investigated the free vibration of a cantilever beam, made of unidirectional fiber-reinforced composite, of high aspect ratio and with an open edge crack is. The beam model is based on the classical lamination theory; the crack modeled with the local flexibility method such that the cantilever beam could be replaced with two intact beams with the crack as the additional boundary condition. It was demonstrated that changes of Eigen-frequencies and corresponding mode shapes depend on not only the crack location and ratio, but also the material properties (fiber orientation, fiber volume fraction).

Kisa (2003), investigated the effects of cracks on the dynamical characteristics of a cantilever composite beam, made of graphite fiber-reinforced polyamide. The finite element and the component-mode synthesis methods were used to model the problem. The cantilever composite beam divided into several components from the crack sections. The effects of the location and depth of the cracks, and the volume fraction and orientation of the fibers on the natural frequencies and mode shapes of the beam with transverse non-propagating open cracks, were explored. The results of the study leaded to conclusions that, presented method was adequate for the vibration analysis of cracked cantilever composite beams, and by using the drop in the natural frequencies and the change in the mode shapes, the presence and nature of cracks in a structure can be detected.

Wang, Inmana and **Farrar** (2004) investigated the coupled bending and torsional vibration of a fiber-reinforced composite cantilever with an edge surface crack. The model was based on linear fracture mechanics, the Castigliano's theorem and classical lamination theory. The crack was modeled with a local flexibility matrix such that the cantilever beam was replaced with two intact beams with the crack as the additional boundary condition. The coupling of bending and torsion can result from either the material properties or the surface crack.

Lu and **Law** (2009) studied such effect from multiple cracks in a finite element in the dynamic analysis and local damage identification. The finite beam element was formulated using the composite element method with a one-member–one-element configuration with cracks where the interaction effect between cracks in the same element was automatically included. The accuracy and convergence speed of the proposed model in computation were compared with existing models and experimental results. The parameter of the crack model was found needing adjustment with the use of the proposed model.

Gaith (2011) implemented a continuous cracked beam vibration theory for the lateral vibration of cracked Euler–Bernoulli beams with single-edge open cracks. In his study, the crack identification for simply supported graphite/epoxy fiber-reinforced composite beams is considered. The effects of crack depth and location, fiber orientation, and fiber volume fraction on the flexibility and consequently on natural frequency and mode shapes for cracked fiber-reinforced composite beams are investigated

2.3 Objective and Scope of the present investigation

The influence of crack on the dynamic characteristics like natural frequencies, modes of vibration of isotropic beam has been the subject of many investigations. However studies related to dynamic behavior of composite beam with crack are scarce in literature.

The present work deals with the vibration analysis of composite beam made from carbon fiber reinforced epoxy with a transverse open crack i.e. V-notch using the finite element software ANSYS.

Chapter 3

MODELLING

&

MODAL ANALYSIS

3.1 Introduction.

ANSYS is commercial finite element software with capability to analyze a wide range of different problems. Like any finite element software, ANSYS solves governing differential equations by breaking the problem into small elements. The FEA software ANSYS includes time-tested, industrial leading applications for structural, thermal, mechanical, computational fluid dynamics and electromagnetic analysis. ANSYS software solves for the combined effects of multiple forces, accurately modeling combined behaviors resulting from "metaphysics interaction".

The ANSYS batch language has many features of the FORTRAN programming language. If statements and do loops can all be included in ANSYS batch files. In addition ANSYS has several built-in functions for further manipulation of ANSYS results or geometry parameters.

There two primary ways to use ANSYS interactively through the graphical user interface and through the use of batch files and ANSYS commands. In this project, we have used the GUI. It is easiest to learn ANSYS interactively, especially when compared to the daunting task of learning all of the relevant ANSYS commands. Interactive ANSYS has disadvantages such as it requires the user to save the model geometry, mesh, and results in a *.db file, which can get as large as 50MB or more and the second one is Interactive use is slow if you need to repeat operations. Meanwhile, the advantages of batch processing include an entire model, mesh, and solution description can be contained in a file of 10-100K. Sub Batch processing is highly modular. If you spend time creating batch files, changing dimensions and mesh densities is a snap.

The finite element simulation was done by finite element analysis package ANSYS 13. This is used to perform the modeling of the composite beam and calculation of natural frequencies with relevant mode shape.

3.2 The Methodology

Modal analysis of ANSYS is used to determine the natural frequencies and mode shapes, which are important parameters in the design of a structure for dynamic loading conditions. They also required for spectrum analysis or for a mode superposition harmonic transient analysis.

Modal analysis in ANSYS program is linear analysis. The mode extraction method includes Block Lanczos (default), sub space, Power Dynamics, reduced, unsymmetric, and damped and QR damped. The damped and QR damped methods allow to include damping in the structure.

3.3 Governing Equation

The differential equation of the bending of a beam with a mid-plane symmetry (B_{ij} = 0) so that there is no bending-stretching coupling and no transverse shear deformation (ε_{xz}=0) is given by;

$$IS_{11}\frac{d^4\omega}{d\omega^4} = q(x) \tag{1}$$

It can easily be shown that under these conditions if the beam involves only a one layer, isotropic material, then $IS_{11} = EI = Ebh^3/12$ and for a beam of rectangular cross-section Poisson's ratio effects are ignored in beam theory, which is in the line with Vinson & Sierakowski (1991).

In Equation 1, it is seen that the imposed static load is written as a force per unit length. For dynamic loading, if Alembert's Principle are used then one can add a term to Equation.1 equal to the product mass and acceleration per unit length. In that case Equation.1 becomes

$$IS_{11}\frac{d^4\omega(x,t)}{d\omega^4}=q\left(x,t\right)-\rho F\frac{\partial^2\omega\left(x,t\right)}{\partial x^2} \tag{2}$$

where ω and q both become functions of time as well as space, and derivatives therefore become partial derivatives, ρ is the mass density of the beam material, and here F is the beam cross- sectional area. In the above, q(*x, t*) is now the spatially varying time-dependent forcing function causing the dynamic response, and could be anything from a harmonic oscillation to an intense one-time impact.

For a composite beam in which different lamina have differing mass densities, then in the above equations use, for a beam of rectangular cross-section,

$$\rho F=\rho bh=\sum_{k=1}^{N}\rho b(h_k-h_{k-1}) \tag{3}$$

However, natural frequencies for the beam occur as functions of the material properties and the geometry and hence are not affected by the forcing functions; therefore, for this study let q(x,t) be zero.

Thus, the natural vibration equation of a mid-plane symmetrical composite beam is given by;

$$IS_{11}\frac{d^4\omega\left(x,t\right)}{d\omega^4}+\rho F\frac{\partial^2\omega\left(x,t\right)}{\partial x^2}=0 \tag{4}$$

It is handy to know the natural frequencies of beams for various practical boundary conditions in order to insure that no recurring forcing functions are close to any of the natural frequencies, because that would result almost certainly in a structural failure. In each case below, the natural frequency in radians /unit time is given as

$$\omega_n = \alpha^2 \sqrt{IS_{11} / \rho F L^4} \tag{5}$$

Where α^2 is the co-efficient, which value is catalogued by Warburton, Young and Felgar and once ω_n is known then the natural frequency in cycles per second (Hertz) is given by $f_n = \omega_n / 2\pi$, which is in the line with Vinson & Sierakowski (1991).

In general, governing equation for free vibration of the beam can be expressed as

$$[K] - \omega^2 [M] \{q\} = 0 \tag{6}$$

Where, K = Stiffness matrix

M = Mass matrix , and

q = degrees of freedom.

3.4 Beam Model

The model chosen is a cantilever composite beam of uniform cross-section A, having an open transverse crack of depth ‘a’ at position L_1. The width, length and height of the beam are B, L and H, respectively in Fig. 3.1. The angle between the fibers and the axis of the beam is α.

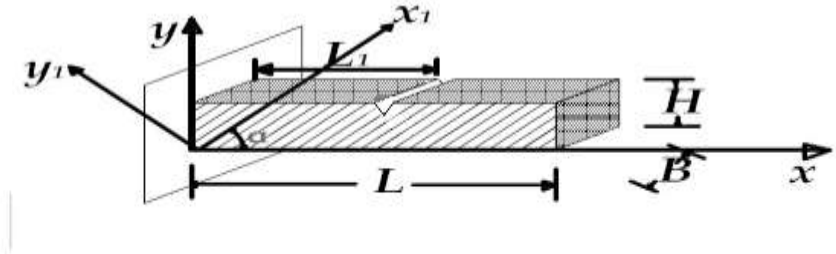

Figure. 3.1 Schematic diagram cantilever composite beam with a crack

3.5 Modelling Procedure in ANSYS 13

Regardless of the type of problem involved, an ANSYS analysis consists of the same steps as follows:

1. Preprocessing
2. Solution stage
3. Post processing.

After selecting the type of analysis in the preferences, the next step in the preprocessing is to choose an element type. The element type includes a list of general categories such as Structural Mass, Structural Link, Structural Solid, Beam, Solid Sell etc. A number of different specific elements will appear for each general category. Each element has its own set of DOFs, which are the degrees of freedom for which ANSYS will find a solution. Next material properties, real constants, section etc. need to input.

The modeling phase entails geometry definition. This is where you draw a 2D or 3D representation of the problem. ANSYS has a very powerful modeler built into the preprocessor.

The modeler allows the user to construct surfaces and solids to model a variety of geometries. For any given geometry, there are often several different ways to create the model. Before the meshing phase you will define material properties and choose a finite element suitable for the problem. In the meshing phase the model discretized i.e. creating the mesh.

In the solution phase, boundary conditions and loads need to be defined. The types of loads and boundary conditions you select depend on the simplifications being made. ANSYS will then attempt to solve the system of equations defined by the mesh and boundary conditions.

Finally, when the solution is complete, you will need to review the results using the post processor. The ANSYS post processor provides a powerful tool for viewing results .These results may be color contour plots, line plots, or simply a list of DOF results for each node.

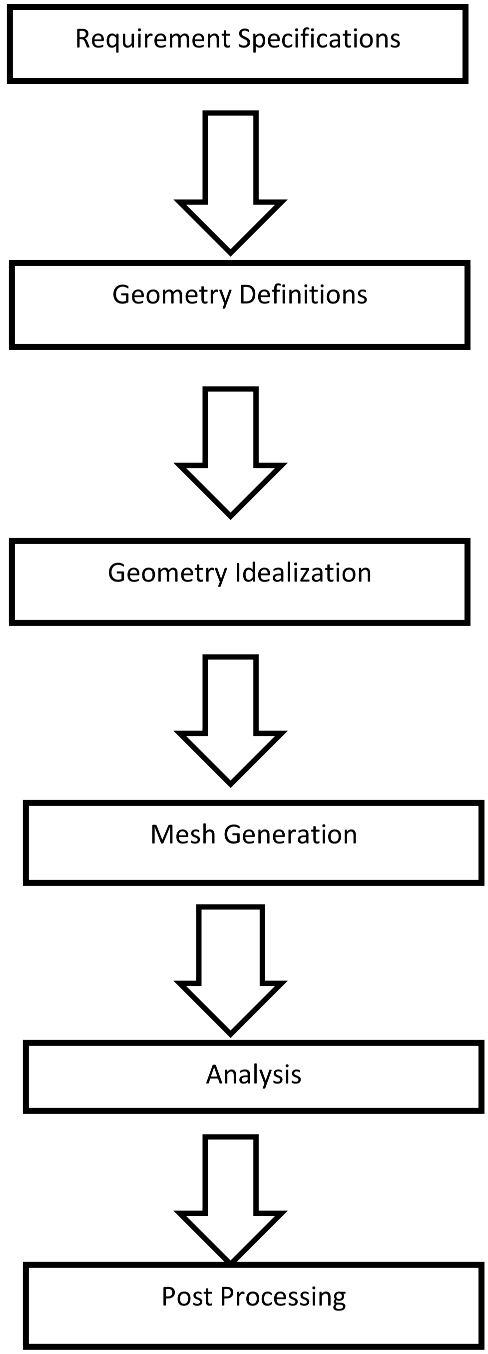

Fig. 3.2 Flow chart for ANSYS

3.6 Element Description:

Solid shell 190 (SOLSH 190)

SOLSH190 is used for layered applications such as modelling laminated shells or sandwich constructions with a wide range of thickness. The element is defined by eight nodes and element has three degrees of freedom at each node: translations in the nodal X, Y, and Z directions.

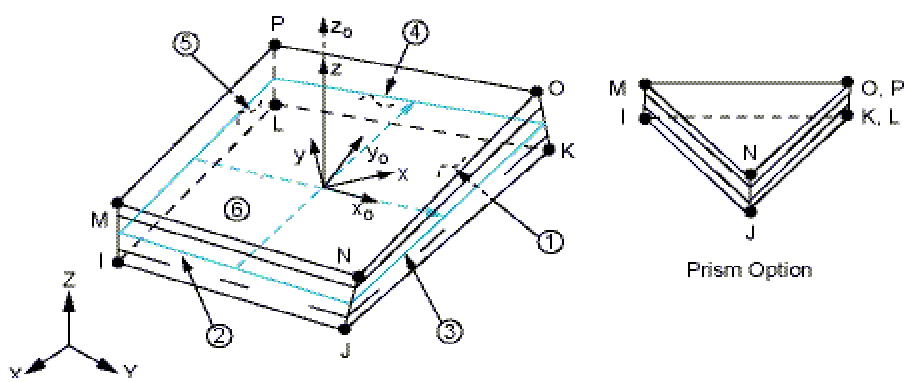

Fig.3.3 SOLSH190 Geometry.

Where x_0 = Element x-axis if ESYS is not supplied.

x = Element x-axis if ESYS is supplied.

The element coordinate system follows the shell convention where the z axis is normal to the surface of the shell. The node ordering must follow the convention that the I-J-K-L and M-N-O-P element faces represent the bottom and top shell surfaces, respectively.

SOLSH190 is fully compatible with 3-D constitutive relations. Compared to classical shell elements that are based on plane stress assumptions, SOLSH190 usually gives more accurate predictions when the shell is thick.

SOLSH 190 Input Data

Nodes

I, J, K, L, M, N, O, P

Degrees of Freedom

U_x , U_y , U_z

Real Constants

None

Material Properties

EX, EY, EZ, PRXY, PRYZ, PRXZ (or NUXY, NUYZ, NUXZ),

ALPX, ALPY, ALPZ (or CTEX, CTEY, CTEZ or THSX, THSY, THSZ),

DENS, GXY, GYZ, GXZ, DAMP

Surface Loads

Pressures

face 1 (J-I-L-K), face 2 (I-J-N-M), face 3 (J-K-O-N),

face 4 (K-L-P-O), face 5 (L-I-M-P), face 6 (M-N-O-P)

Body Loads

Temperatures

T1, T2, T3, T4, T5, T6, T7, T8 for 8 element nodes. Temperatures at layer interface corners are computed by interpolating nodal temperatures.

Special Features

Plasticity, Hyper elasticity, Viscoelasticity, Large deflection, Large strain etc.

3.7 Meshing:

Meshing a model can be the most difficult part of using any finite element package. While ANSYS gives the user a variety of automatic options so far as meshing is concerned, you are urged to use caution when using these tools. It is usually best to think about how you would like to mesh your model before you even go about making a model and creating areas. In general, ANSYS has two methods of meshing:

1. Free meshing

The free mesh has no recognizable pattern and no regularity in the element shapes. Free meshing is easy but for complex geometries can often lead to distorted elements that undermine accuracy. Too often users free mesh a model because it is easy without bothering to worry about the resulting mesh. Free meshing is available for 2D quadrilateral and triangular element shapes. Free meshing can only produce 3D tetrahedral elements for solid models.

2. Mapped meshing.

Mapped meshes are easier to control and are oftentimes more accurate. Mapped meshes allow the user to more carefully specify the size and shape of the mesh in local regions. Mapped meshing is available for 2D and 3D elements.

Mapped meshes are controlled by specifying element divisions on boundaries and by splitting areas and volumes in certain ways. Once you have split the areas and/or volumes in accordance with the above rules, use **lsel** to select the lines and **lesize** to set the number of element divisions along that line. The feature of mapped meshing allows the user to place smaller elements in the areas of high stress gradient (near the crack) while using larger elements where the gradient is not so steep.

There are restrictions to the use of mapped meshing;

For 2D element, each area must be four-sided i.e. be made up of four lines. If the area is made up of more lines, you will need to split up the area to create sub-areas with four sides or you must concatenate lines so that four lines define the area.

For 3Delement, each volume must have 6 faces (6 bounding areas). You will need to split volumes or concatenate lines and areas to create 6-faced volumes.

3.8 Modeling of composite beam using ANSYS 13

3.8.1. Preferences

Firstly it is required to give preference for what type analysis you want to do, here we are analyzing for beam so structural part is selected as Shown in fig.

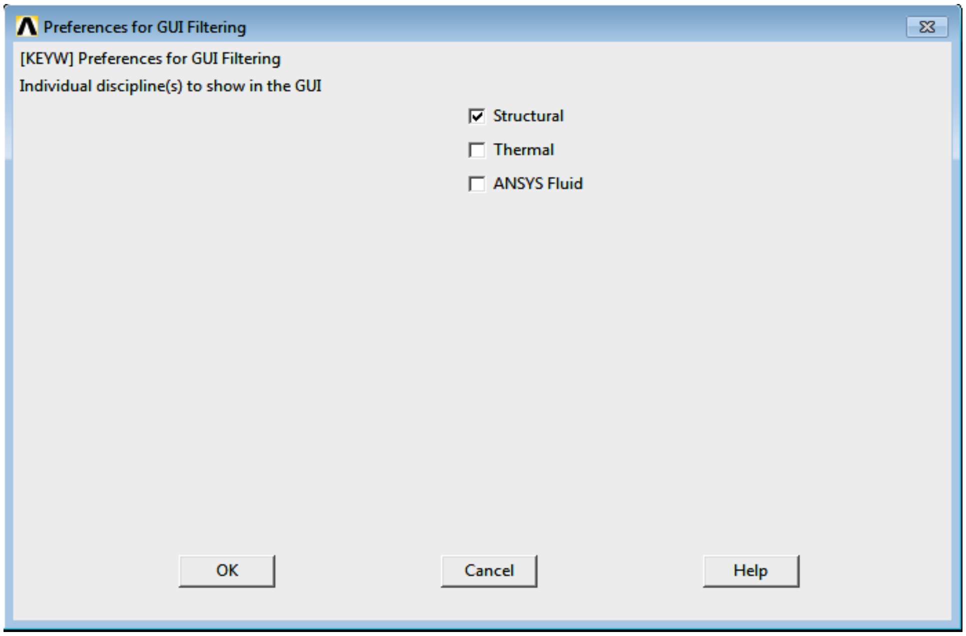

Fig.3.4 Preference menus on ANSYS

3.8.2. Preprocessor

Now next step is preprocessor, where the preprocessor menu basically used to inputs the entire requirement thing for analysis such as- element type, real constraints, material properties, modeling, meshing, and loads. Element menu contains – defined element type and degree of freedom defined where we have to give the element structure type such as BEAM, SOLID, SHELL, and the degree of freedom , here in this analysis selected part is SOLID SHELL190 fig . shows this menu contained part –

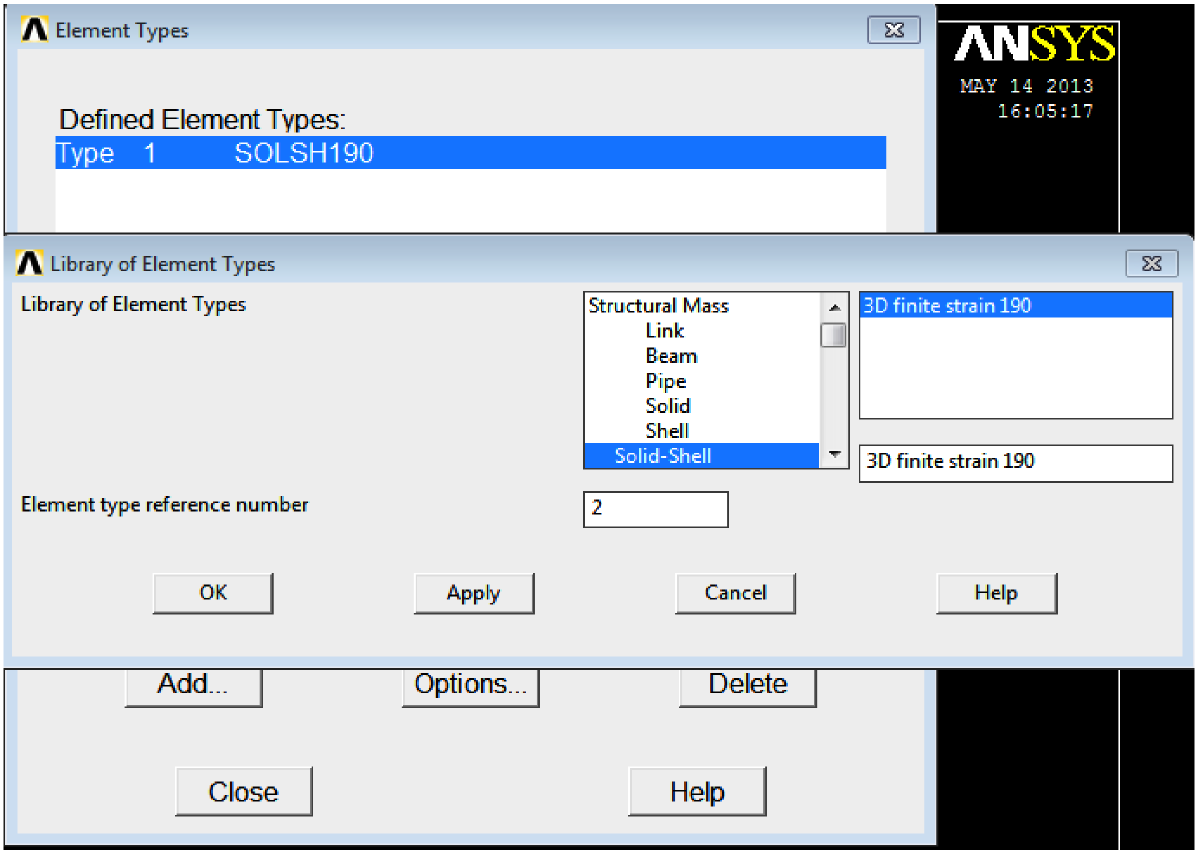

Fig.3.5 Defined element type

The next step is to define material properties, by the help of material model menu in the ANSYS13 as shown in figure below.

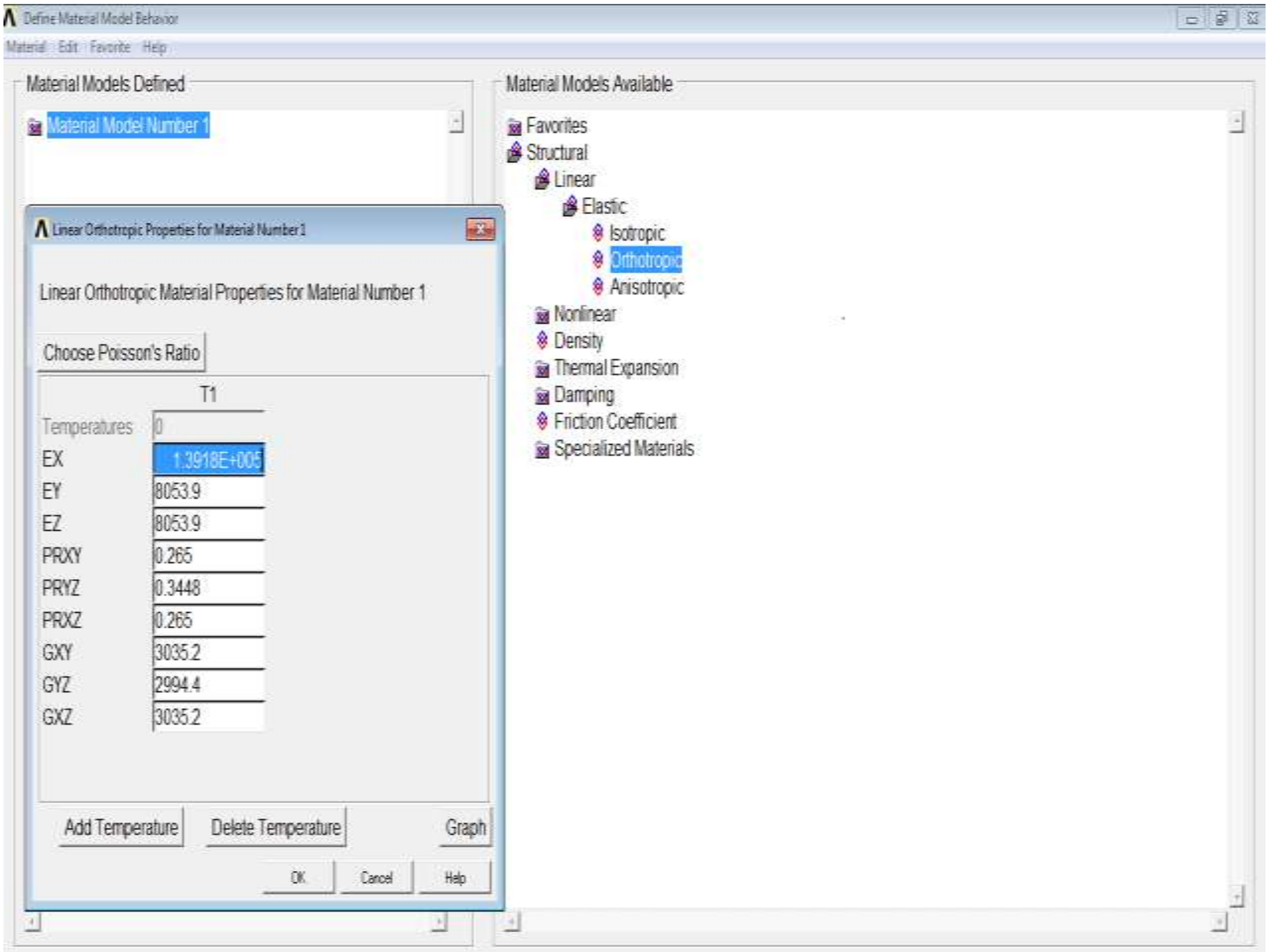

Fig. 3.6 Define material model behaviors

For material model behavior considered material as orthotropic and for this given all the nine input which is required (young modulus, modulus of rigidity and Poisson ratio). After this the next step is to create shell section, the layered composite specifications including layer thickness, material, orientation, and number of integration points through the thickness of the layer are specified via shell section commands as shown in fig.11

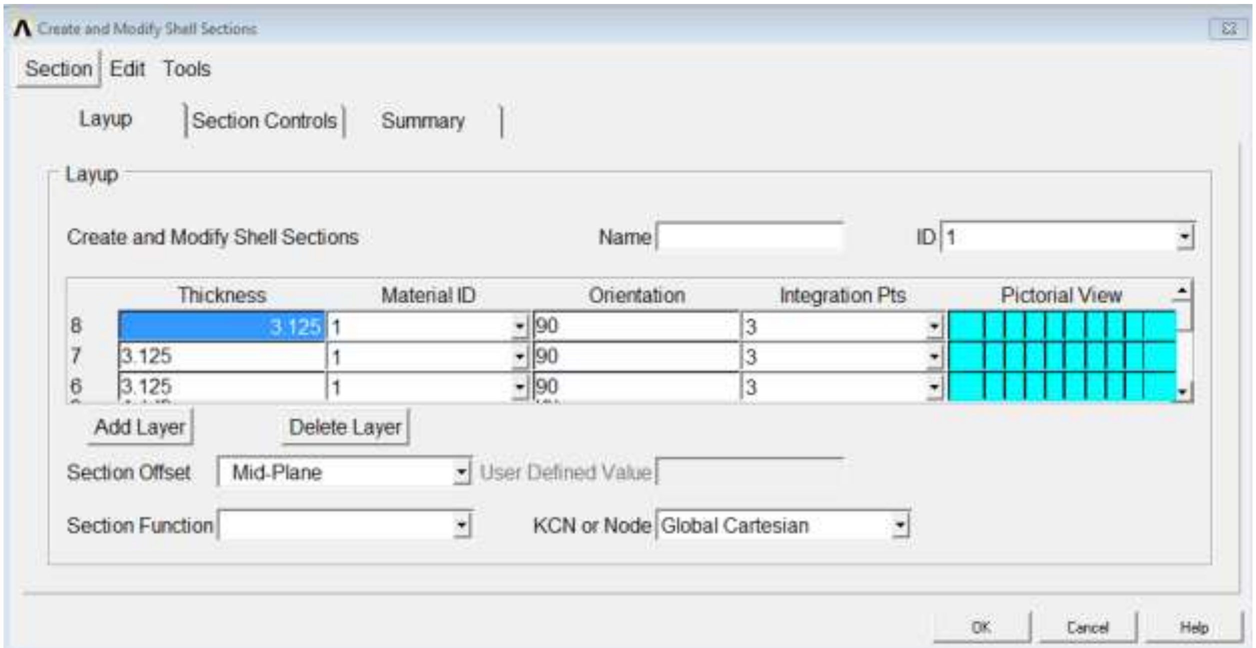

Fig.3.7 Create and Modify shell section.

You can designate the number of integration points (1, 3, 5, 7, or 9) located through the thickness of each layer. Two points are located on the top and bottom surfaces respectively and the remaining points are distributed equal distance between the two points. The element requires at least two points through the entire thickness. When no shell section definition is provided, the element is treated as single-layered and uses two integration points through the thickness

Next step is to modeling the model given in numerical problems to analyze, means the geometry shape of the structure, in this analysis we considered the shape of beam is rectangular cross section. The key steps in creating the model are as follows:

1. Key points:

    i. Go to **Main Menu → Preprocessor → Modelling →Create →Keypoints**

→ **In Active CS**.

ii. In the box; write 0,0,0. This will create a key point at the origin.

iii. Click **Apply.**

iv. Repeat steps 3 and 4 to create the required points in the geometry.

v. Click **Ok**.

2. Lines :

Join all the key points with line , which make it as line element.

i. Go to **Create → Lines → Straight lines**.

Pick and join all key points using straight lines.

ii. In order to view lines properly, go to **Plot →lines**.

3. Areas:

Here we will convert the line element to area element to create our required geometry.

i. Go to **Create → Areas → Arbitrary → By lines**.

ii. Select all lines, click **Ok**.

In order to create a volume element, now we will extrude the area element.

4. Extrude:

i. Go to **Operate→Extrude →Areas→Along Normal**

ii. Give the length of extrusion i.e., width of the beam and click **Ok**.

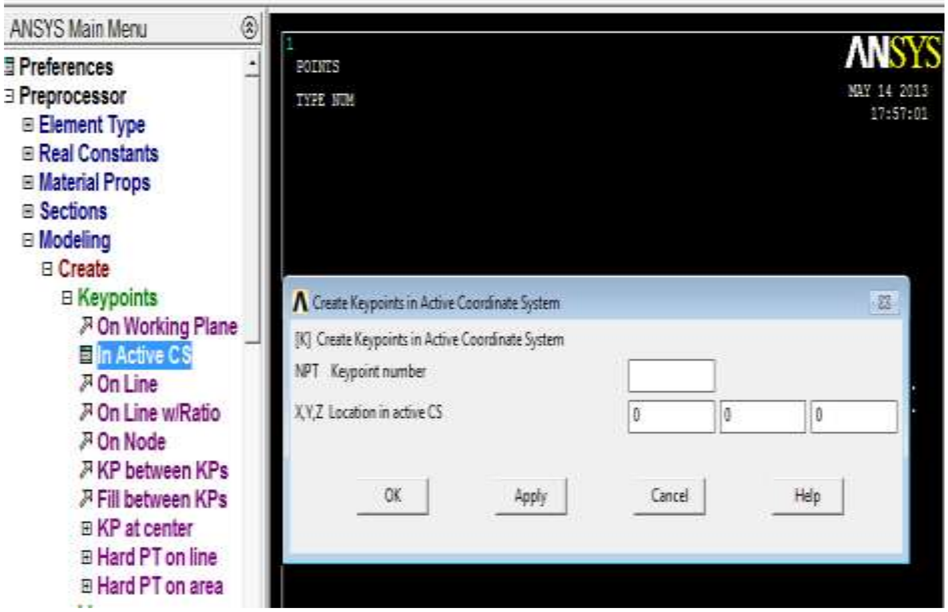

Fig.3.8 Create key points in active co-ordinate system

After the completion of this stage now meshing menu is introduced to mesh the problem. How to mesh in the ANSYS is showing in fig. . The step to follow the mesh is given as:

1. Go to **Preprocessor → Meshing → Mesh tool → Size controls → Global → Set**.
2. Set **Size Element Edge Length** to 4. This will create a mesh of square elements with width 4 units.
3. Click OK.
4. Select **Mesh →Volumes → Hex → Sweep**
5. Click **Sweep** and select the beam for meshing.

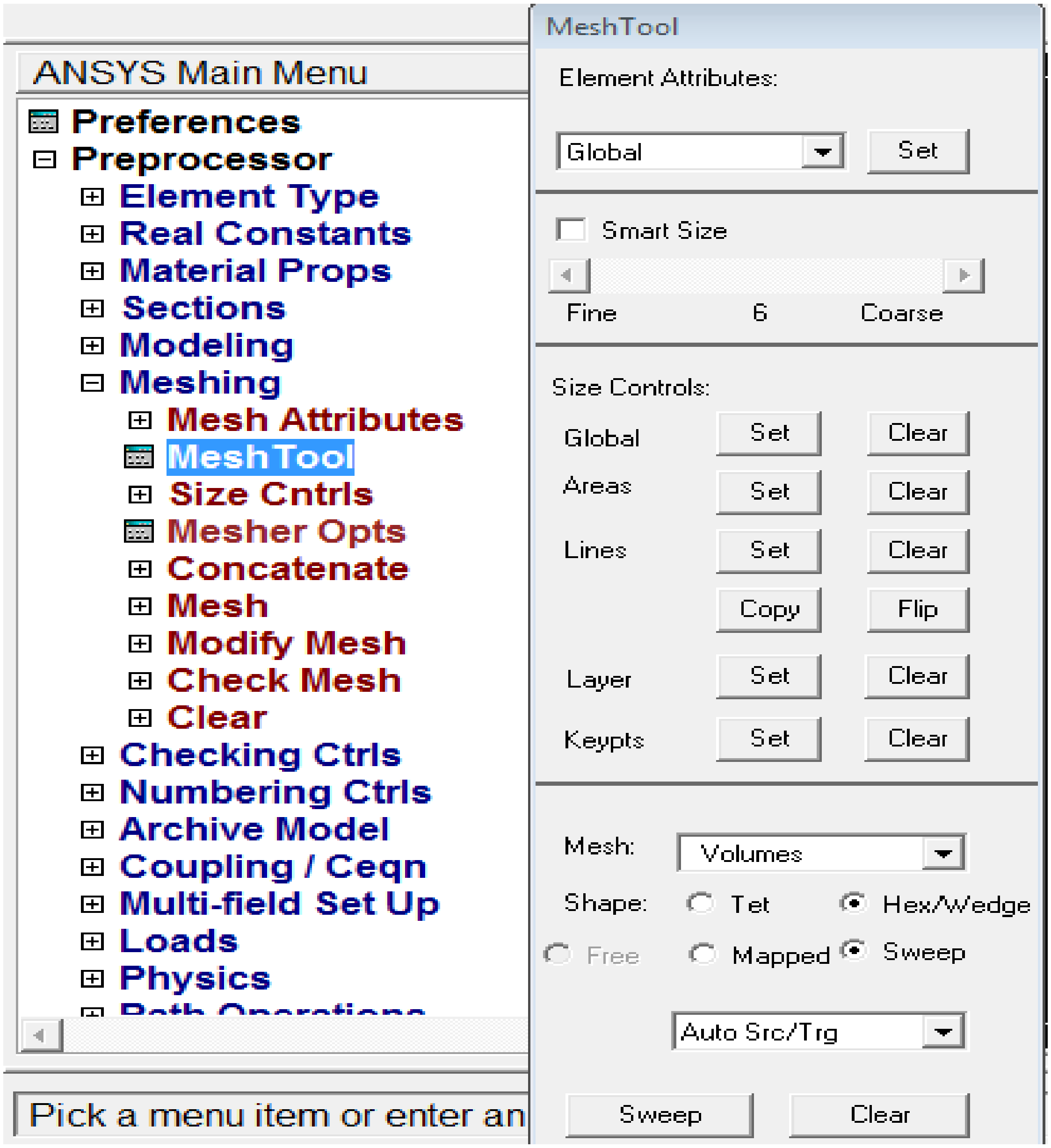

Fig.3.9 Mesh tool

The next step is to define the boundary condition for the composite beam modeled. The following steps will describe how to impose clamped-free boundary to the beam:

1. Go to **Main menu → Preprocessor → Loads → Define loads → Apply → Structural**

→ **Displacement** → **on Areas**.

2. Select the area to be restrained and click **Ok**.
3. Click **ALL DOF** to secure all degrees of freedom.
4. Under **Value Displacement value** put ‘0’. The selected area is now a fixed end.
5. Click **Ok.**

3.8.3. Solution Stage

The default direct frontal solver is fine for small linear problems. However, the size limitations become obvious when the user attempts to solve large 3D problems. Solving the FE problem is tantamount to solving a matrix equation with a very large matrix. Iterative methods are generally faster for bigger problems. ANSYS provides several different solver options, each of which may be more or less appropriate for a given problem.

Before going to solve the problem, we have to introduce the analysis type i.e., static analysis, harmonic analysis, or modal analysis etc. to the problems. We have selected modal analysis for our problem, because we want frequencies and mode shapes as output.

Following steps are to be followed for type of analysis:

1. Go to **Preprocessor** →**Loads** → **Analysis type** → **New analysis**.
2. Click **Modal** in the type of analysis box.
3. Go to **Analysis options** → **Mode extraction method** → **Block Lanczos** .
4. Enter the Number of Modes required and click **Ok**

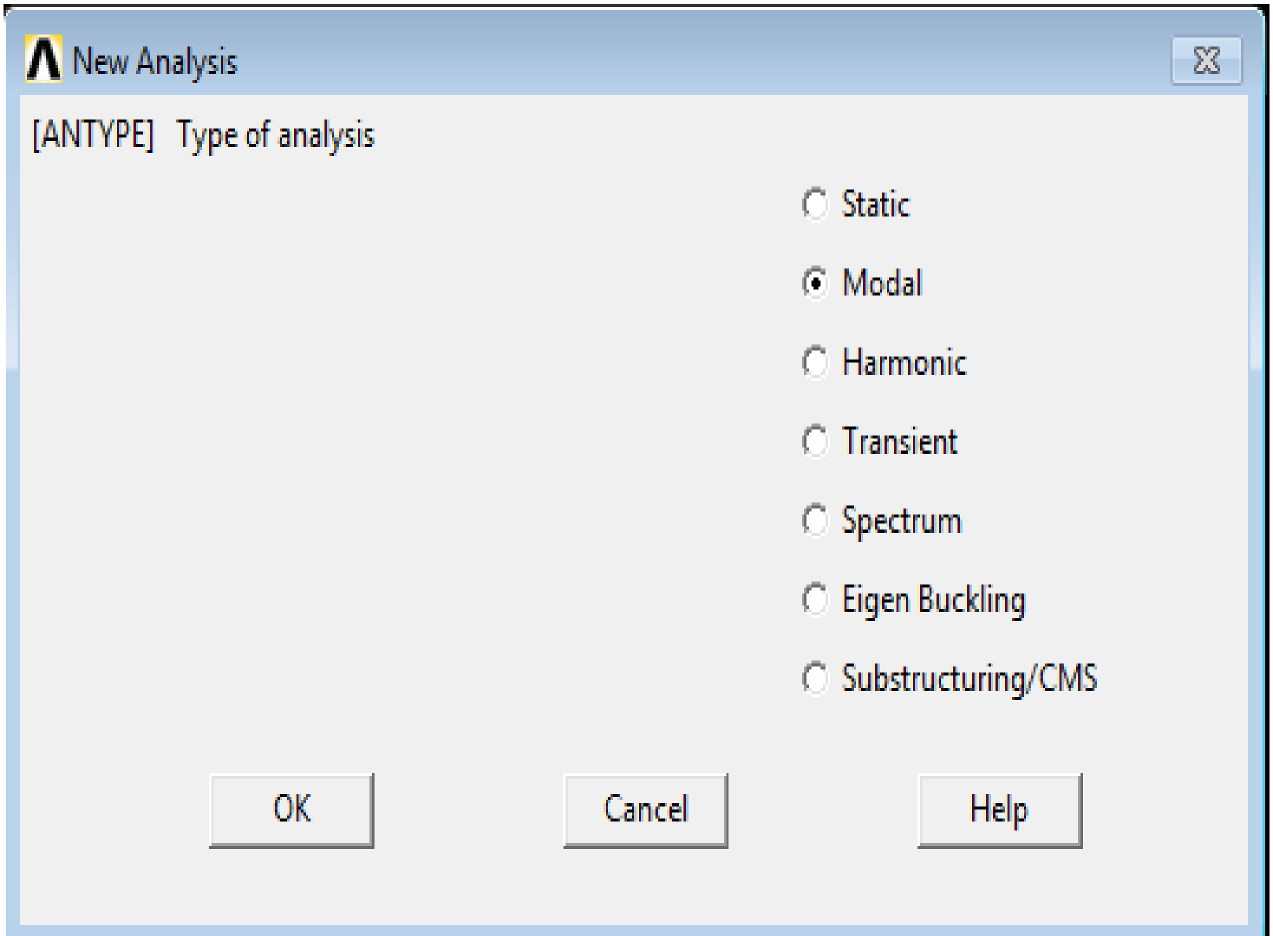

Fig. 3.10 New analysis

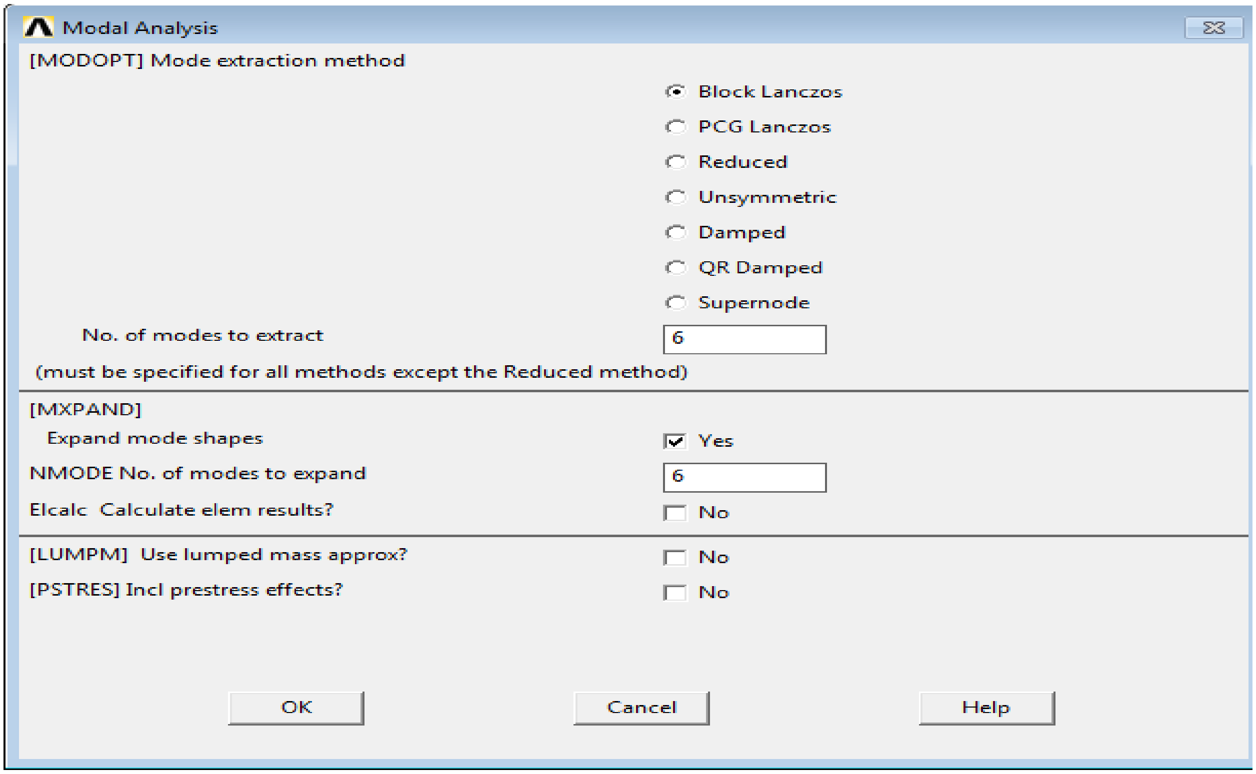

Fig.3.11 Modal analysis

The solution can be obtained by following steps:

1. Go to **Main menu → Solution → Solve → Current LS**
2. Click **Ok**

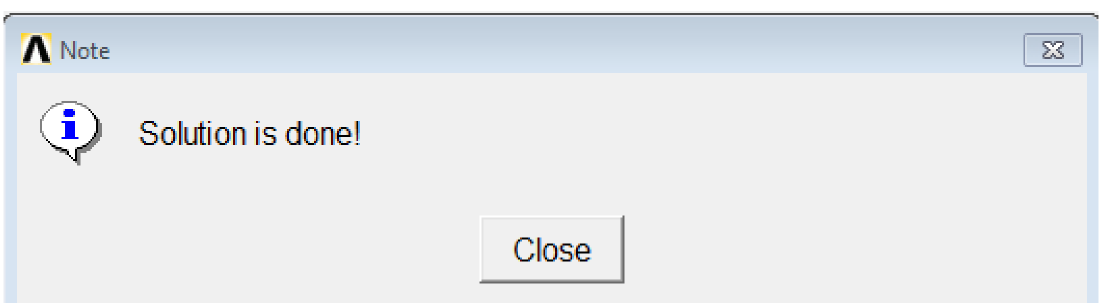

Fig.3.12 Solution Status.

3.8.4. Post Processing

The ANSYS post processor provides a powerful tool for viewing results. We can see the following results in Post processing:

1. Result summary
2. Failure criteria
3. Plot results
4. List results
5. Nodal calculation.

The procedure for vibration analysis of composite beam with crack, i.e., V-notch is same as the above described steps, but a little bit modification in modelling process. Crack is created in the composite beam using key points and lines to define areas. Volumes can be made from extruding the areas and then using Boolean operation to achieve a crack in a composite beam. For proper meshing, we will divide the beam at location of crack into two volumes using working plane.

Chapter 4

RESULTS & DISCUSSION

4.1 Introduction

Effect of crack and various other parameters on the dynamic characteristics of a composite beam are studied and compared with previously studied results.

In order to check the accuracy of the present analysis, the case considered in Krawczuk & Ostachowicz (1995) is adopted here. The beam assumed to be made of unidirectional graphite fiber-reinforced polyamide. The geometrical characteristics and material properties of the beam are chosen as the same of those used in Krawczuk & Ostachowicz (1995). The material properties of the graphite fiber-reinforced polyamide composite, in terms of fibers and matrix, is identified by the indices f and m, respectively, are in Table-4.1

Table.4.1 Properties of the graphite fibre-reinforced polyamide composite

Modulus of Elasticity	E_m	2.756 GPa
	E_f	275.6 GPa
Modulus of Rigidity	G_m	1.036 GPa
	G_f	114.8GPa
Poisson's Ratio	ν_m	0.33
	ν_f	0.2
Mass density	ρ_m	1600 kg/m^3
	ρ_f	1900 kg/m^3

The geometrical characteristics, the length (L), height (H) and width (B) of the composite beam, are taken as 1.0 m, 0.025 m and 0.05m respectively.

In this chapter, the results of vibration analysis of composite beam structure with or without crack are presented. Each of the cracked composite beam problems is presented separately for the following studies:

I. Comparison with Previous Studies

II. NumericalResults

A. Vibration Analysis of composite beam with single crack

B. Vibration Analysis of composite beam with multiple cracks

4.2 Comparison with Previous Studies.

Quantitative results on the effects of various parameters on the vibration analysis of intact and cracked composite are presented.

4.2.1 Vibration analysis studies

The presented method has been applied for the free vibration analysis of an intact and cracked composite cantilever beam. Free vibration analysis of a cantilever cracked composite beam has been examined by Krawczuk & Ostachowicz (1995) using finite element method (FEM). In this study the results obtained with present element are compared with the results of Krawczuk & Ostachowicz. In addition, the three lowest natural frequencies for various values of the orientation of fiber (α) and the fiber volume fraction (V) are determined and given in Table-4.2 and Fig. 4.1, 4.2. In Fig..4.3 and 5.4 the changes of the two first natural frequencies of the beam due to the crack as functions of the angle of fibers (α) are compared with the results of Krawczuk & Ostachowicz (1995). As seen from the tables agreements are good. The non-dimensional natural frequencies are normalized according to the following relation;

$$\omega_n(\alpha) = L\sqrt{\omega(\alpha) / \sqrt{S_{11} I / \rho F}} \qquad (7)$$

Where ω(α) is the natural frequency of the beam computed for each value of the angle of fibers(α).

Table.4.2 Comparison of First three non-dimensional natural frequencies of the non- cracked composite beam as a function of the angle of fibers α.(For volume fraction of fiber V =0.1 and 0.3)

Angle of Fibers (deg.)	Volume Fraction of fibers (V)	Present Analysis			Krawczuk (1995)		
		1st Non-dimensional Nat. freq	2nd Non-dimensional Nat. freq	3rd Non-dimensional Nat. freq	1st Non-dimensional Nat. freq	2nd Non-dimensional Nat. freq	3rd Non-dimensional Nat. freq
0	0.1	1.8669	4.6273	7.6261	1.8701	4.6202	7.6803
15		1.8243	4.5300	7.4841	1.8176	4.5147	7.5141
30		1.665	4.1530	6.9033	1.6545	4.1294	6.8968
45		1.454	3.6353	6.0678	1.3802	3.5901	6.0901
60		1.208	3.0230	5.0513	1.1537	3.0158	5.0178
75		1.099	2.7514	4.5973	1.0813	2.7452	4.5704
90		1.081	2.7026	4.5340	1.0801	2.7102	4.5171
0	0.3	1.877	4.6113	7.5073	1.8514	4.5282	7.6489
15		1.818	4.4873	7.3447	1.8176	4.4447	7.3737
30		1.648	4.0982	6.7804	1.6545	4.0294	6.9268
45		1.388	3.4682	5.7818	1.3899	3.4332	5.8571
60		1.106	2.7684	4.6260	1.1537	2.7158	4.7664
75		0.9480	2.3713	3.9632	1.0813	2.2705	4.0403
90		0.930	2.3263	3.8831	1.0801	2.2172	3.9762

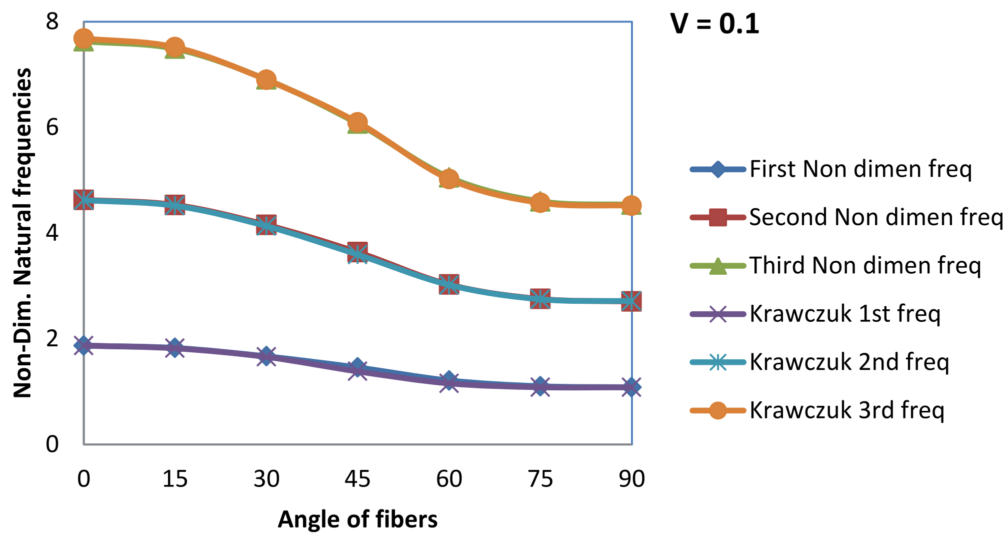

Fig.4.1. First three non-dimensional frequencies of the non-cracked composite beam a function of the angle of fibers α and for V: 0.1

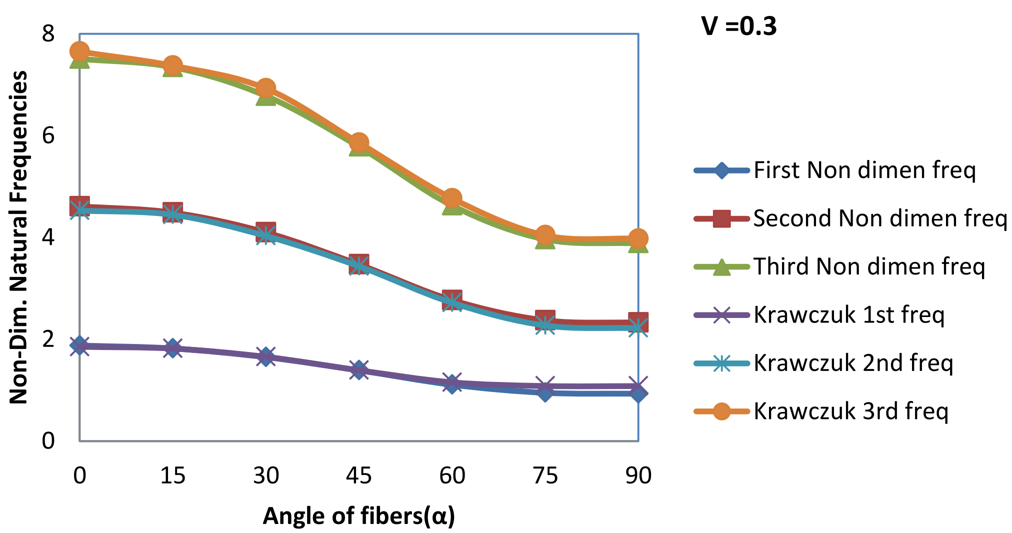

Fig.4.2. First three non-dimensional frequencies of the non-cracked composite beam as a function of the angle of fibers α and for V: 0.3

Table 4.3. Comparison of non-dimensional natural frequencies of the composite beam with crack as a function of the angle of fibers (α) for crack depth a/H =0.2, 0.4, 0.6 (volume fraction of fiber V = 10%, crack location L1/L = 0.1).

Angle of Fibers (degrees)	Relative Crack depth (a/H)	Present Analysis		Krawczuk(1995)	
		1st Non-dimensional Nat. freq	2nd Non-dimensional Nat. freq	1st Non-dimensional Nat. freq	2nd Non-dimensional Nat. freq
0	0.2	1.7201	4.600	1.7100	4.5400
15		1.7360	4.5656	1.7260	4.5600
30		1.7755	4.6064	1.7705	4.6000
45		1.8442	4.620	1.8300	4.6600
60		1.8538	4.6771	1.8528	4.6700
75		1.8600	4.6708	1.8590	4.6711
90		1.8638	4.6735	1.8700	4.6800
0	0.4	1.4420	4.4200	1.420	4.410
15		1.4628	4.4590	1.4658	4.4500
30		1.5359	4.5075	1.5452	4.5000
45		1.7301	4.598	1.7100	4.5700
60		1.7939	4.6243	1.7940	4.6232
75		1.8032	4.6389	1.8172	4.6492
90		1.8288	4.6528	1.8400	4.6600
0	0.6	1.1540	4.1020	1.1200	4.220
15		1.2530	4.2414	1.2930	4.2400
30		1.3484	4.3243	1.3184	4.3300
45		1.3970	4.4470	1.4300	4.4600
60		1.6511	4.5332	1.6523	4.5400
75		1.7189	4.5533	1.7189	4.5600
90		1.7377	4.5749	1.7700	4.6000

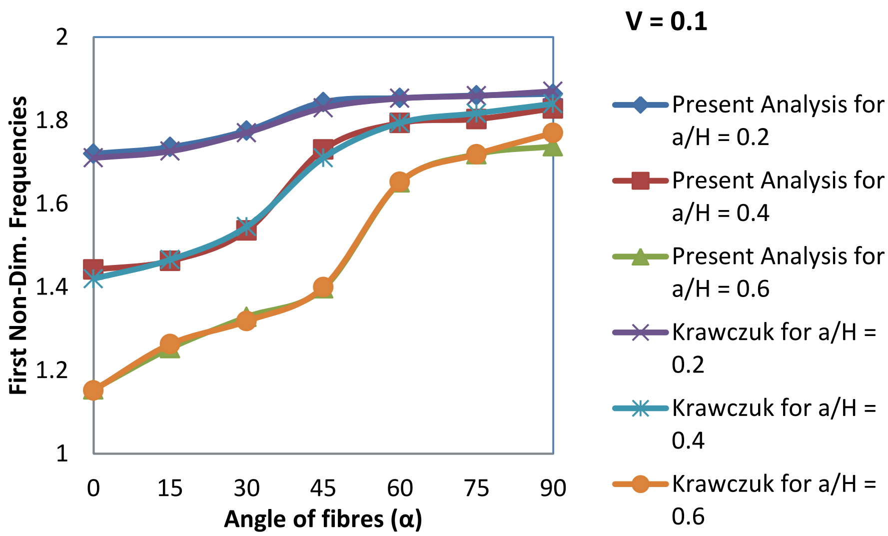

Fig.4.3 First non-dimensional natural frequencies of the cracked composite beam as a function of the angle of fibers (α) for different values of the crack depth a/H = 0.2, 0.4 and 0.6 (V = 0.1, crack location L1/L = 0.1)

From fig. 4.1 and 4.2 it can be seen that the natural frequencies of composite beam decreases as the angle of fiber orientation increases. After fiber angle 70 degree the change in all the three frequencies are unaffected. Therefore it can be concluded that the natural frequencies of composite beam is a function of angle of fibers.

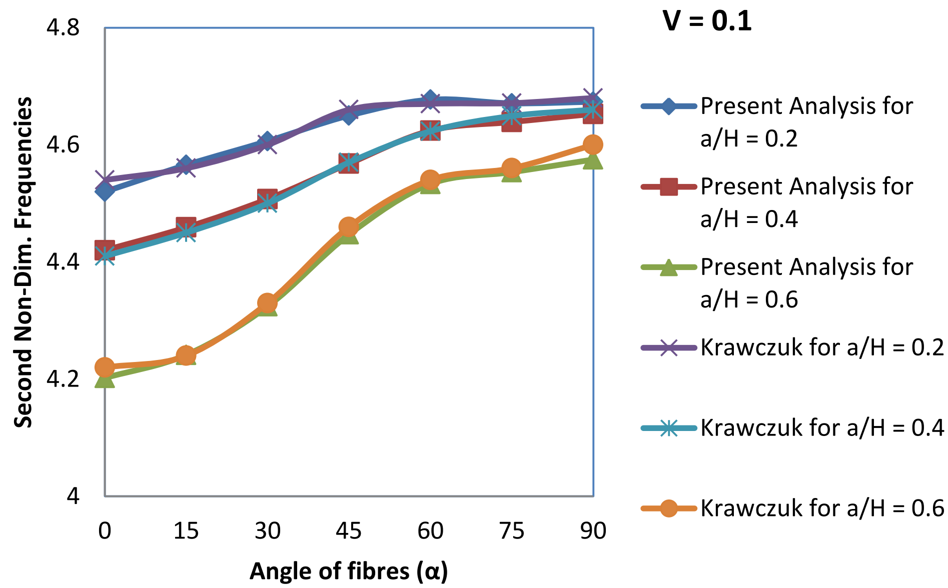

Fig.4.4 Second non-dimensional natural frequencies of the cracked composite beam as a function of the angle of fibers (α) for different values of the crack depth a/H = 0.2, 0.4 and 0.6 (V = 0.1, crack location L1/L = 0.1)

From fig.4.3 and 4.4 it can be depicted that in case of beam with crack, the natural frequencies increases as the angle of fiber orientation increases. The most difference in frequency occurs when angle of fibers is zero degree. This is due to the fact that the flexibility of the composite beam due to crack is a function of the angle between the crack and the reinforcing fibers.

4.3 Numerical Results

After obtaining the comparison with previous study with the existing literatures, the results for various parametric studies like effect of geometry, crack location and support conditions on natural frequencies of composite beam are presented. The changes of the two first natural frequencies of the beam due to the crack as functions of fiber volume fraction are analyzed. Similarly, the three first natural frequencies of the composite beam due to the crack as functions of fiber orientations (α) and fiber volume fractions are analyzed for free vibration of a composite beam with multiple cracks for different crack positions. The beam assumed to be made of unidirectional graphite fiber-reinforced polyamide. The geometrical characteristics of the graphite fiber-reinforced polyamide composite beam are chosen as the same of those used in Krawczuk & Ostachowicz (1995). The material properties of the graphite fiber-reinforced polyamide composite are taken as below:

E_{11} = 139.18GPa,

E_{22} = 8.0539GPa,

G_{12} = 3.0352GPa,

G_{23} = 2.9944GPa,

ν_{12} = 0.2650,

ν_{23} = 0.3448,

The geometrical characteristics, the length (L), height (H) and width (B) of the composite beam were chosen as 1.0m, 0.025 and0.050 m, respectively.

4.3.1(A) Vibration Analysis of composite beam with single crack

Effect of various parameters like volume fraction of fibers, length of the beam, boundary conditions and crack locations of cracked composite beam on first, second and third non-dimensional natural frequencies studied and explained as below.

(a) Effect of Volume Fraction of fiber on Natural frequencies

Table 4.4 First non-dimensional natural frequencies of the cracked composite beam as a function of volume fraction of fibers V for different values of the crack depth a/H = 0.2, 0.4 and 0.6 (angle of fibers α = 0 degree and crack location L1/L= 0.1)

Angle of fibers(degree)	Volume fraction of fiber	First Non-dimensional Frequency	
		Relative crack depth	Relative crack depth
0	0	1.7749	1.6767
	0.1	1.7437	1.5542
	0.3	1.7107	1.4945
	0.5	1.7090	1.4912
	0.7	1.7254	1.5189
	0.9	1.7653	1.5943
	1	1.8049	1.7019

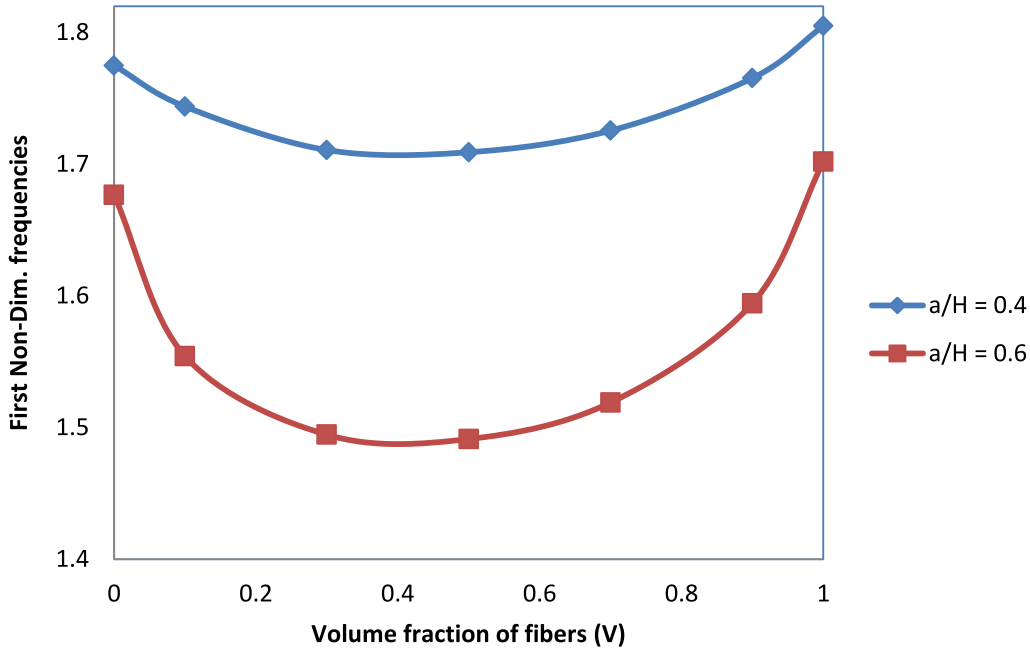

Fig. **4.5** First non-dimensional natural frequencies of the cracked composite beam as a function of volume fraction of fiber V for different values of the crack depth a/H = 0.2, 0.4 and 0.6 (angle of fibers α = 0 degree, crack location x/L = 0.1)

Table.4.5 Second non-dimensional natural frequencies of the cracked composite beam as a function of volume fraction of fibers V for different values of the crack depth a/H = 0.2, 0.4 and 0.6 (angle of fibers α = 0 degree and crack location L1/L= 0.1)

Angle of fibers(degree)	Volume fraction of fiber	Second Non-dimensional Frequency	
		Relative crack depth	Relative crack depth
0	0	4.5167	4.4458
	0.1	4.5356	4.4402
	0.3	4.4851	4.3930
	0.5	4.4828	4.3911
	0.7	4.5110	4.4168
	0.9	4.5690	4.4745
	1	4.5969	4.5240

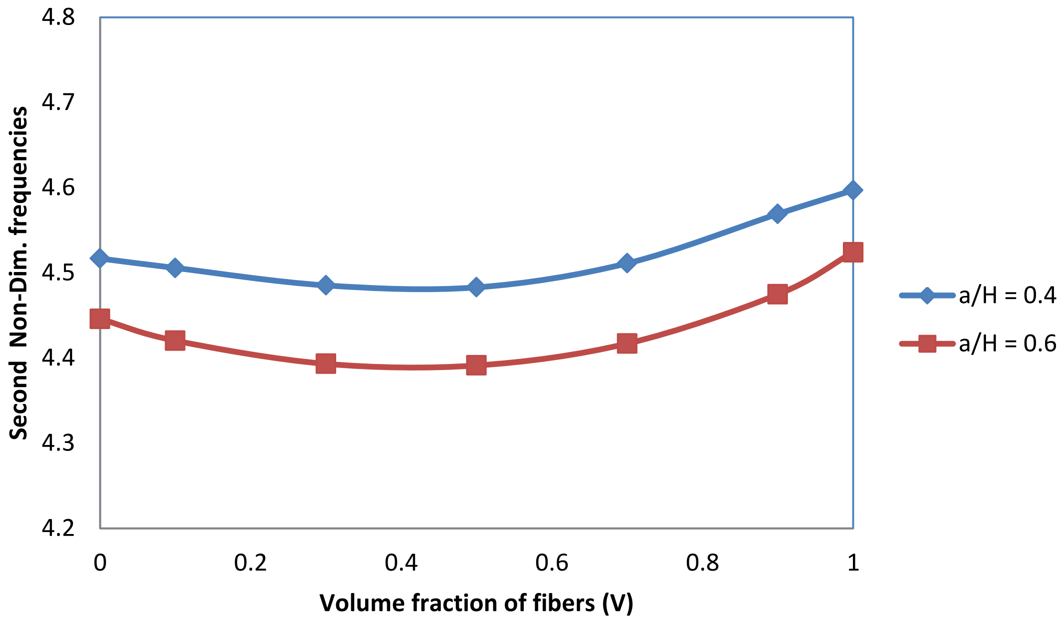

Fig.4.6 Second non-dimensional natural frequencies of the cracked composite beam as a function of volume fraction of fiber V for different values of the crack depth a/H = 0.2, 0.4 and 0.6 (angle of fibers α = 0 degree, crack location x/L = 0.1)

Fig.4.5 and 4.6 presents the influence of the volume fraction of fibers on the first two non-dimensional natural frequencies for different values of the crack depth ratios (a/H). Here various values of the fiber volume fraction have been considered to study its effect on first and second non dimensional frequencies. The angle of fiber is taken as zero degree and crack location is at a distance L1= 0.1L (m) from the fixed end of the beam. The flexibility due to crack is high when the volume fraction of the fiber is between 0.2 and 0.8 and maximum when the fiber fractions is nearly 0.45. This is due to the fact that the flexibility of the composite beam due to crack is a function of fiber volume fraction. Therefore, if the fiber volume fraction is between 0.2 and 0.8

and crack depth ratio is getting higher, the frequency reductions are relatively high as observed in above graphs.

(b) Effect of Length of beam on Natural frequencies

Table 4.6 Natural frequencies of the intact composite beam as a function of the angle of fiber α 0 for different length (volume fraction of fibers V = 50%).

Length of the Beam (m)	Natural Frequencies		
	First natural frequency	Second natural frequency	Third natural frequency
0.5	3.6987	8.6740	13.4613
1	1.8671	4.5871	7.4687
1.5	1.2470	3.0950	5.1109
2	0.9354	2.3312	3.8713
2.5	0.3675	0.9198	1.5383

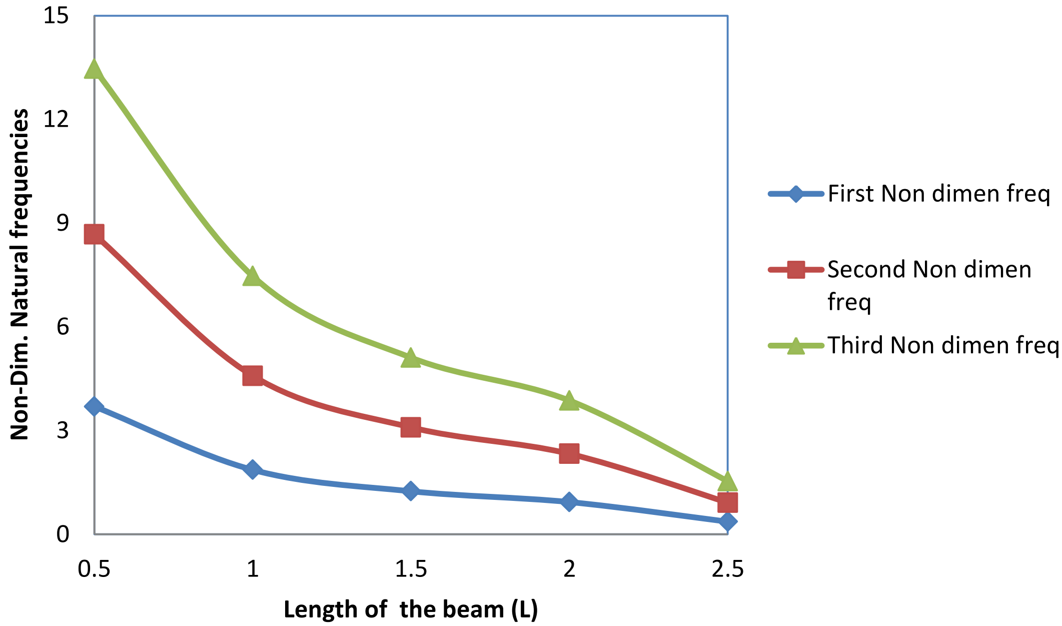

Fig.4.7 First three non-dimensional frequencies of the intact composite beam as a function of length of beam

Table 4.7 Natural frequencies of the composite beam with crack as a function of the angle of fiber α = 0 for different length (volume fraction of fibers V = 50%).

Length of the Beam (m)	Natural Frequencies		
	First natural frequency	Second natural frequency	Third natural frequency
0.5	3.1896	8.4656	13.4366
1	1.7088	4.4828	7.4480
1.5	1.1712	3.0383	5.0967
2	0.8916	2.2962	3.8609
2.5	0.3627	0.9148	1.5342

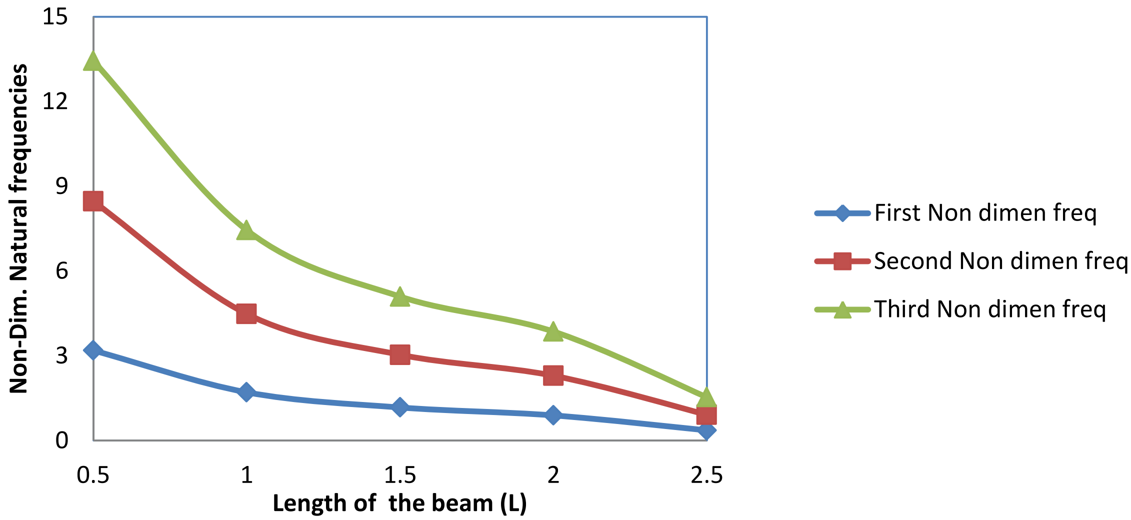

Fig. 4.8 First three non-dimensional frequencies of the cracked composite beam as a function of length of beam (L). (for V= 0.5 and a = 0.4)

Fig. 4.7 and 4.8 shows how the length of beam influence the first three non dimensional frequencies for volume fraction of fiber V = 0.5 and crack depth ratio a/H = 0.4. it is observed from figures that as length composite beam increases , the non-dimensional natural frequencies decreases for both intact and cracked beam cases. This is because as we know that natural frequency is directly proportional to square root of stiffness of the beam and the stiffness of beam is inversely proportional to cube of length.

(c) Effect of Support conditions on Natural frequencies

Table 4.8 Natural frequencies of the composite beam as a function of the angle of fiber α for different boundary condition (for volume fraction of fibers V = 50%).

Angle of fiber (α)	Clamped- Free			Clamped-clamped		
	1st natural frequency	2nd natural frequency	3rd natural frequency	1st natural frequency	2nd natural frequency	3rd natural frequency
0	1.8671	4.5871	7.4687	4.5744	7.3390	9.8903
45	1.3212	3.3009	5.5072	3.2508	5.3791	7.5013
90	0.9188	2.2966	3.8342	2.3138	3.8310	5.3446

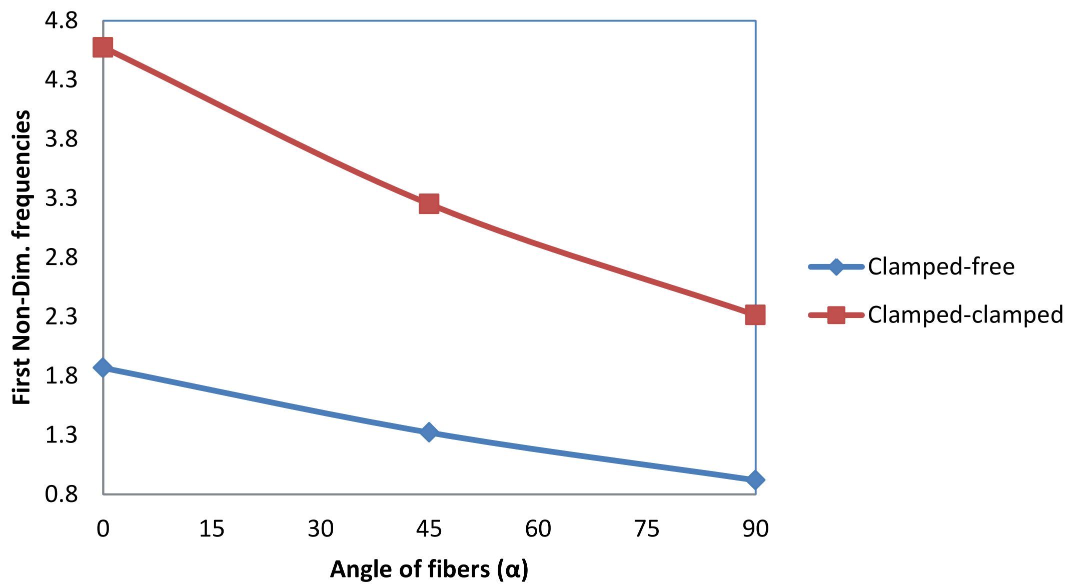

Fig.4.9 First non-dimensional natural frequencies of the intact composite beam as function of angle of fibers α for different support conditions (for V = 0.5)

Table4.9 Natural frequencies of the composite beam with crack as a function of the angle of fiber α for different boundary conditions (for volume fraction of fibers V = 50%, a/H= 0.4)

Angle of fiber (α)	Clamped- Free			Clamped-clamped		
	1st natural frequency	2nd natural frequency	3rd natural frequency	1st natural frequency	2nd natural frequency	3rd natural frequency
0	0.8977	2.2762	3.8228	2.2960	3.8235	5.3415
45	1.2517	3.1703	5.3297	3.2751	5.5090	7.7318
90	1.8299	4.6473	7.8122	4.6822	7.8130	10.9176

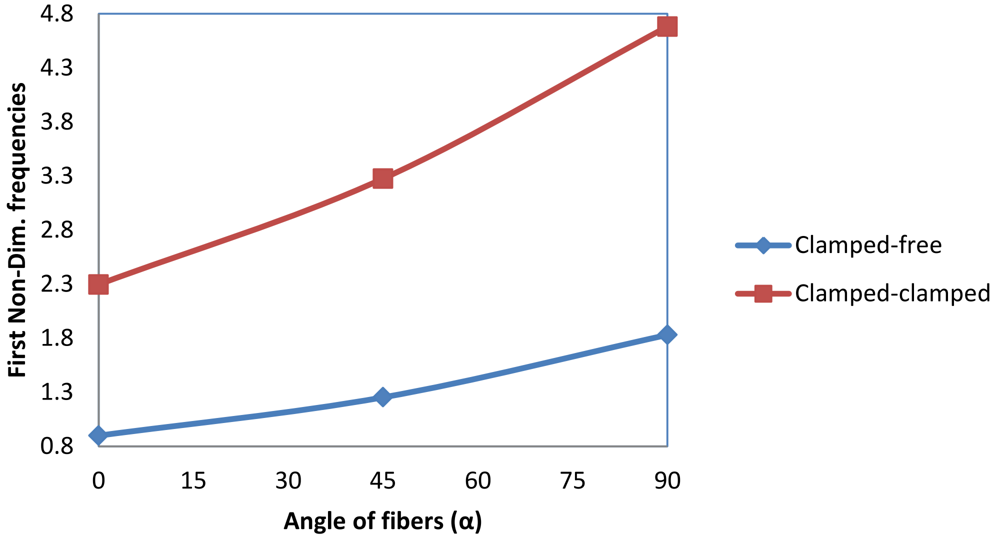

Fig.4.10 First non-dimensional natural frequencies of the cracked composite beam as a function of angle of fibers α for different support conditions (for V = 0.5)

As seen in fig. 4.10 the first non-dimensional natural frequencies are more in case of clamped-clamped support condition than clamped-free support condition. While the natural frequencies decreases as angle of fiber increases for both cases of intact composite beam. But in fig.4.11 the natural frequencies increases for both support conditions because of crack. So one can say that boundary conditions have a remarkable influence on the natural frequencies. The natural frequencies for the clamped-clamped support are higher than Clamped-free support.

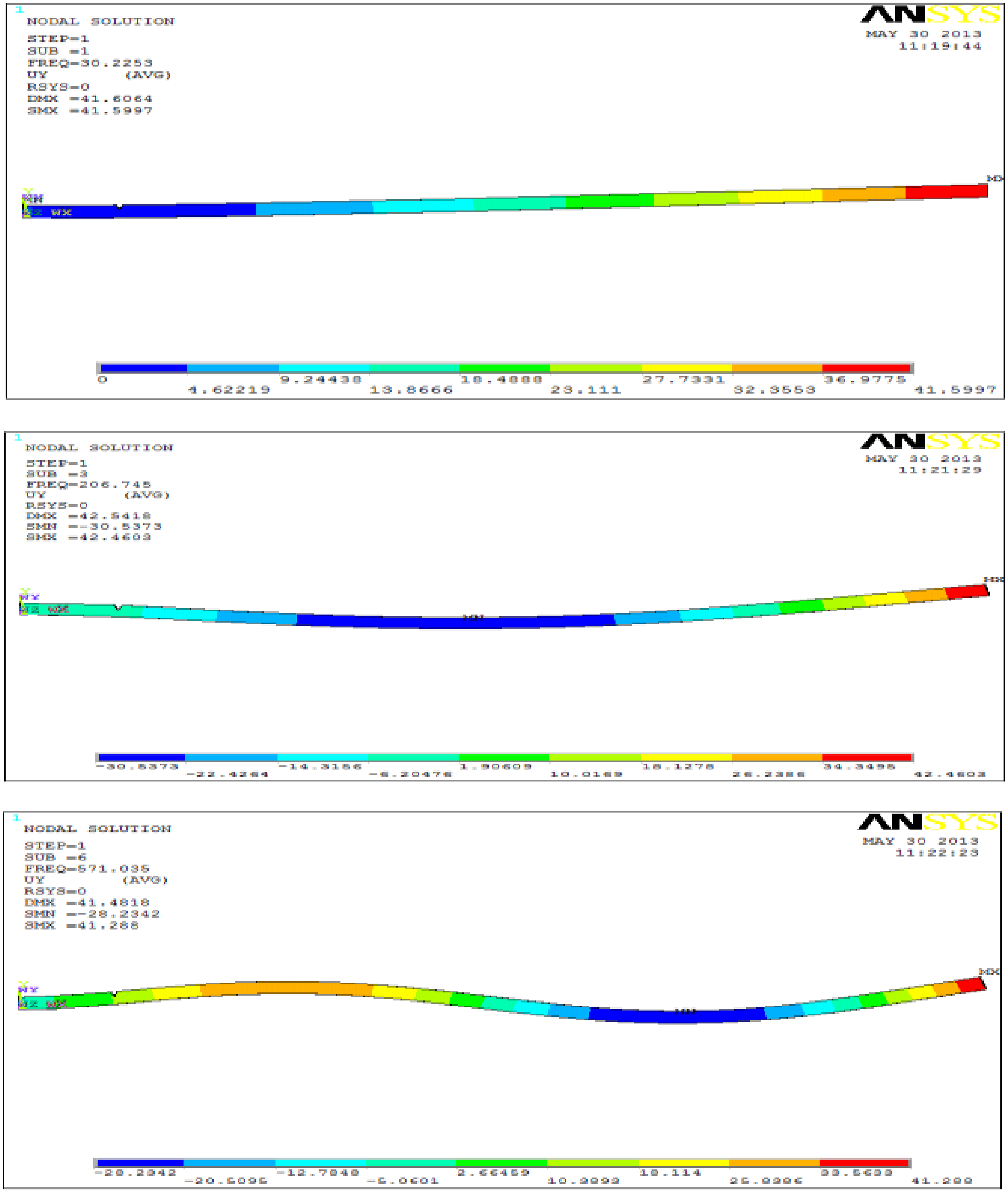

Fig. 4.11 First three mode shapes of cantilever composite beam with crack location at L1/L=0.1

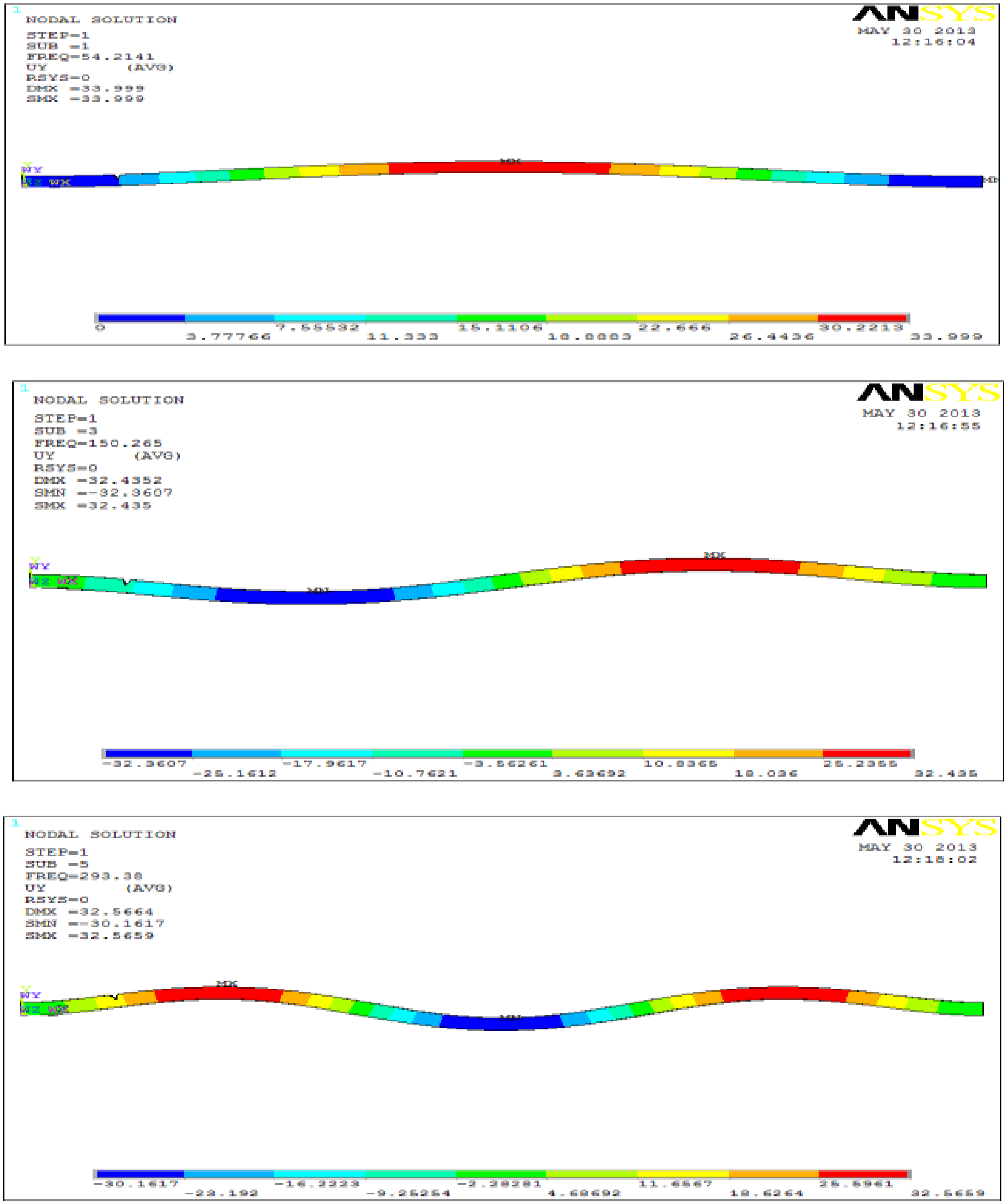

Fig. 4.12 First three mode shapes of clamped-clamped composite beam with crack location at L1/L=0.1

(d) Effect of Crack locations on Natural frequencies

Table 4.10 First non-dimensional natural frequencies of the composite beam with crack as a function of crack location (L1/L) for different crack depth ratios (for volume fraction of fibers V = 0.5)

Angle of fibers(degre	Crack location	First Non-dimensional Frequency		
		Relative crack depth	Relative crack depth	Relative crack depth
0	0	1.8671	1.8671	1.8671
	0.1	1.8268	1.709	1.4912
	0.3	0.9177	0.9082	0.8992
	0.5	0.9057	0.8994	0.8887
	0.7	0.9299	0.9185	0.9178
	0.9	1.8722	1.8693	1.8675

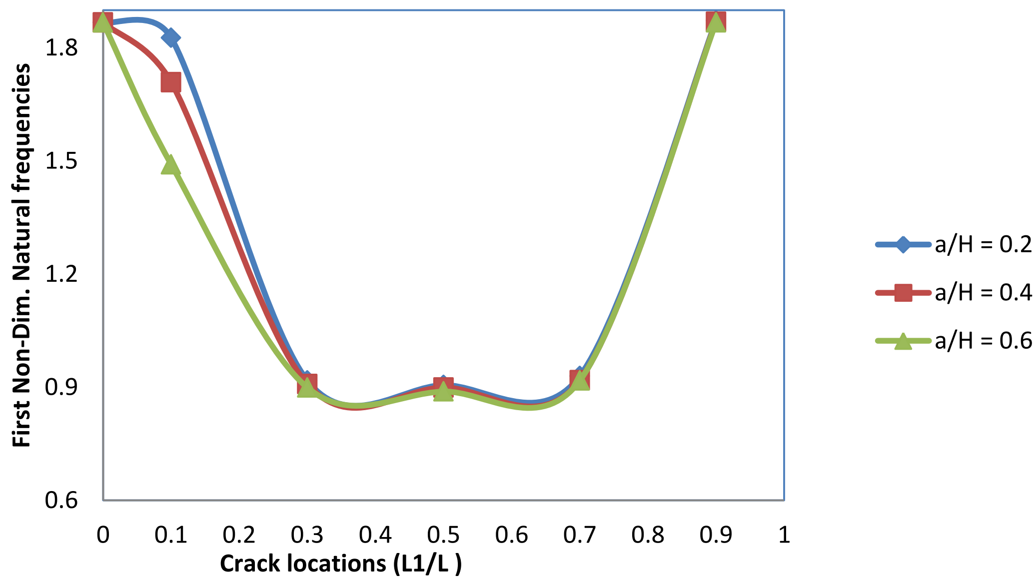

Fig. 4.13 First non-dimensional natural frequencies of the cracked composite beam as a function of different crack location for different crack depth ratios. (for V = 0.5)

Table 4.11 Second non-dimensional natural frequencies of the composite beam with crack as a function of crack location (L1/L) for different crack depth ratios (for volume fraction of fibers V = 0.5

Angle of fibers(degree)	Crack location (L1/L)	Second Non-dimensional Frequency		
		Relative crack depth	Relative crack depth	Relative crack depth
0	0	4.5871	4.5871	4.5871
	0.1	4.5537	4.4828	4.3911
	0.3	2.2957	2.2867	2.2757
	0.5	4.0722	4.0399	3.987
	0.7	2.3407	2.3156	2.298
	0.9	4.5862	4.5839	4.5727

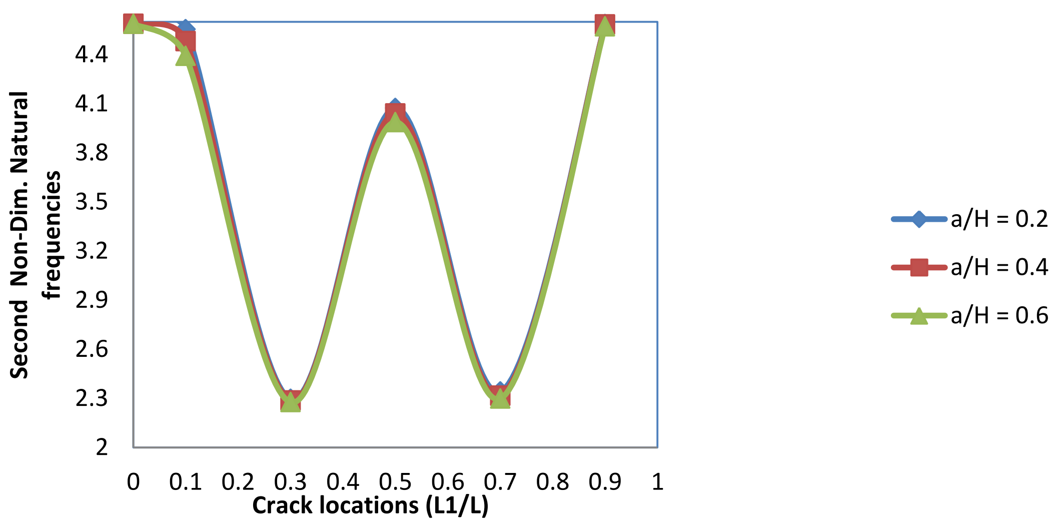

Fig.4.14 Second non-dimensional natural frequencies of the cracked composite beam as a function of different crack location for different crack depth ratios. (for V = 0.5)

From Fig. 4.13, when V_f = 0.5, a/H = 0.2, 0.4 and 0.6 with crack locations (L1/L) from 0 to 0.9 for cantilever composite beam, the first natural frequency is maximum at crack locations L1/L = 0.1 and L1/L = 0.9 . The natural frequency decreases from crack location L1/L = 0.1 up to the minimum value at crack location L1/L = 0.5 and then increases to the maximum value at crack location L1/L = 0.9. The second natural frequency as shown in Figure 4.14 is maximum at crack locations L1/L = 0.1 and L1/L = 0.9 and minimum at crack locations L1/L = 0.3 and L1/L = 0.7.The natural frequency decreases from crack location (L1/L) of zero up to the minimum value at crack locationL1/L = 0.3 and then increases to the maximum value at crack location L1/L = 0.5. The curve is symmetric around the middle crack position (L1/L = 0.5). As the crack depth increases, the corresponding natural frequencies decrease for each crack location. This is compatible with the increase of flexibility or decrease in the stiffness of the beam.

5.1.3 (B) Vibration analysis of composite beam with multiple cracks

The effects of various parameters on the vibration of composite beam with multiple cracks are presented below. The Finite element analysis is carried out for free vibration of a composite cracked beam for various crack locations and crack depth ratio a/H = 0.4 for the example problem considered by Kisa (2003). In Fig. 4.15 to 4.17, the variations of the first three lowest natural frequencies of the composite beam with multiple cracks are shown as a function of fiber orientation (α) for different cracks locations. In these figures three cases, labeled as E, F and G, were considered .The cracks locations (L1/ L, L2 /L, L3/L) for the cases E, F and G, where chosen as (0.05, 0.15, 0.25), (0.45, 0.55, 0.65), (0.75, 0.85, 0.95) respectively. The non-dimensional natural frequencies are normalized according to Eq. (7).

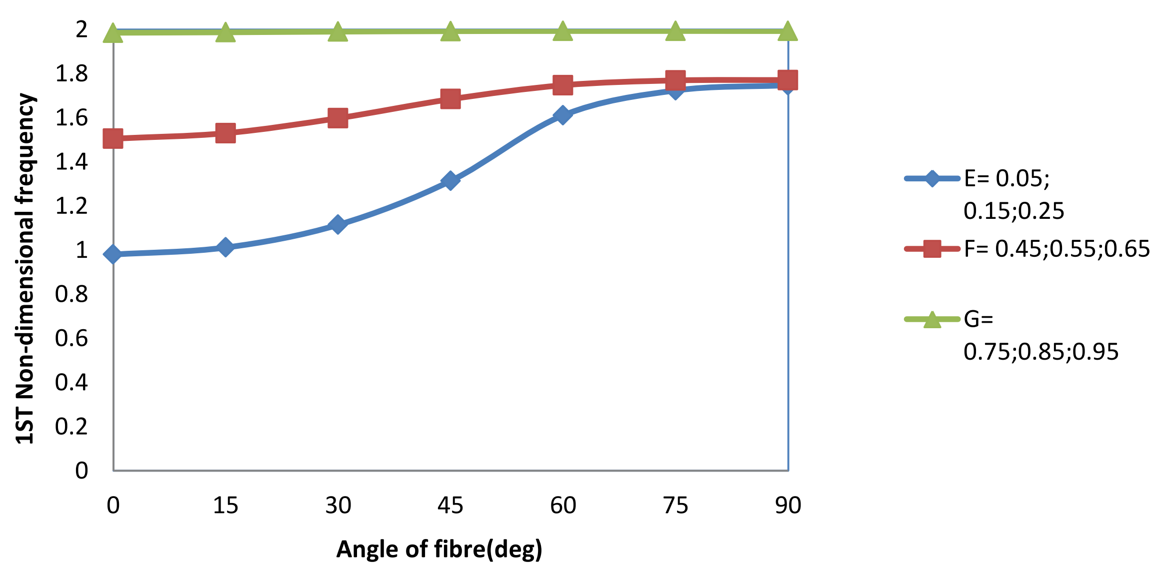

Fig 4.15 The first non-dimensional natural frequencies as a function of angle of fibers for the cases of three cracks located differently, as indicated a/H=0.4 and V=0.5

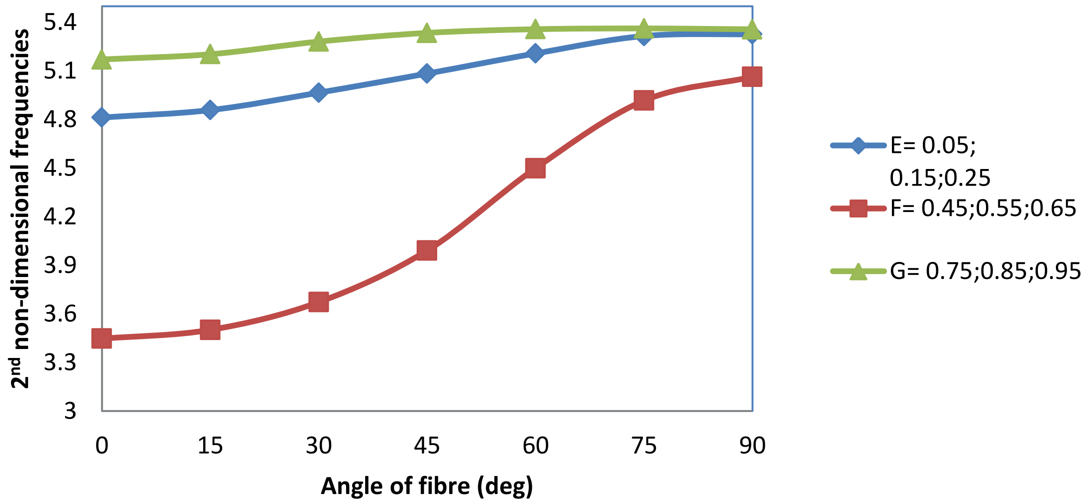

Fig 4.16 The Second non-dimensional natural frequencies as a function of angle of fibers for the cases of three cracks located differently, as indicated a/H=0.4 and V=0.5

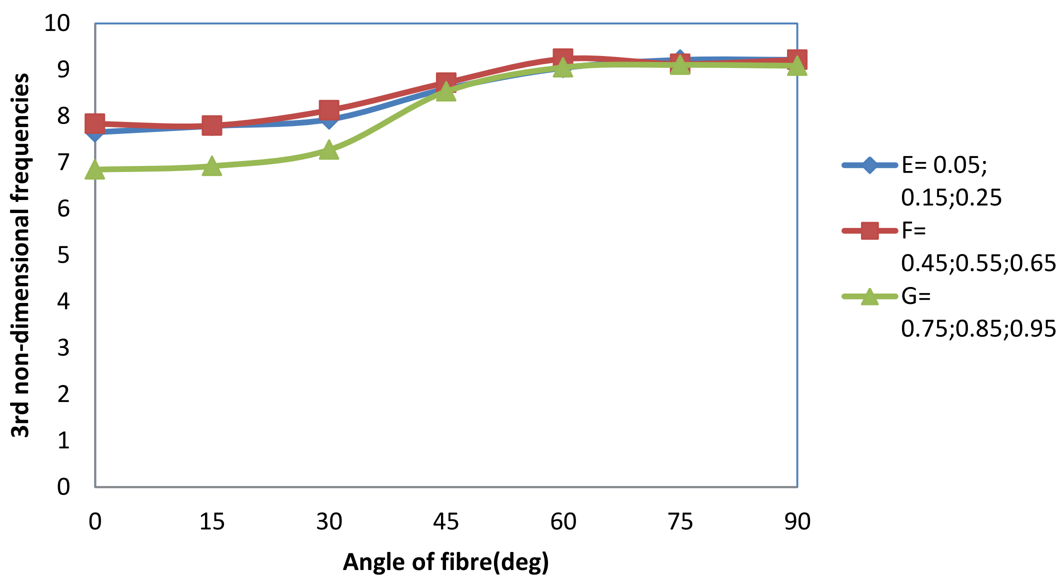

Fig 4.17 The third non-dimensional natural frequencies as a function of angle of fibers for the cases of three cracks located differently, as indicated a/H=0.4 and V=0.5

It can be clearly seen from the Fig 4.15 to 4.17. that, when the cracks are placed near the fixed end the decrease in the first natural frequencies are highest, whereas, when the cracks are located near the free end, the first natural frequencies are almost unaffected. This observation goes to the conclusion that, the first, second and third natural frequencies are most affected when the cracks located at the near of the fixed end, the middle of the beam and the free end, respectively.

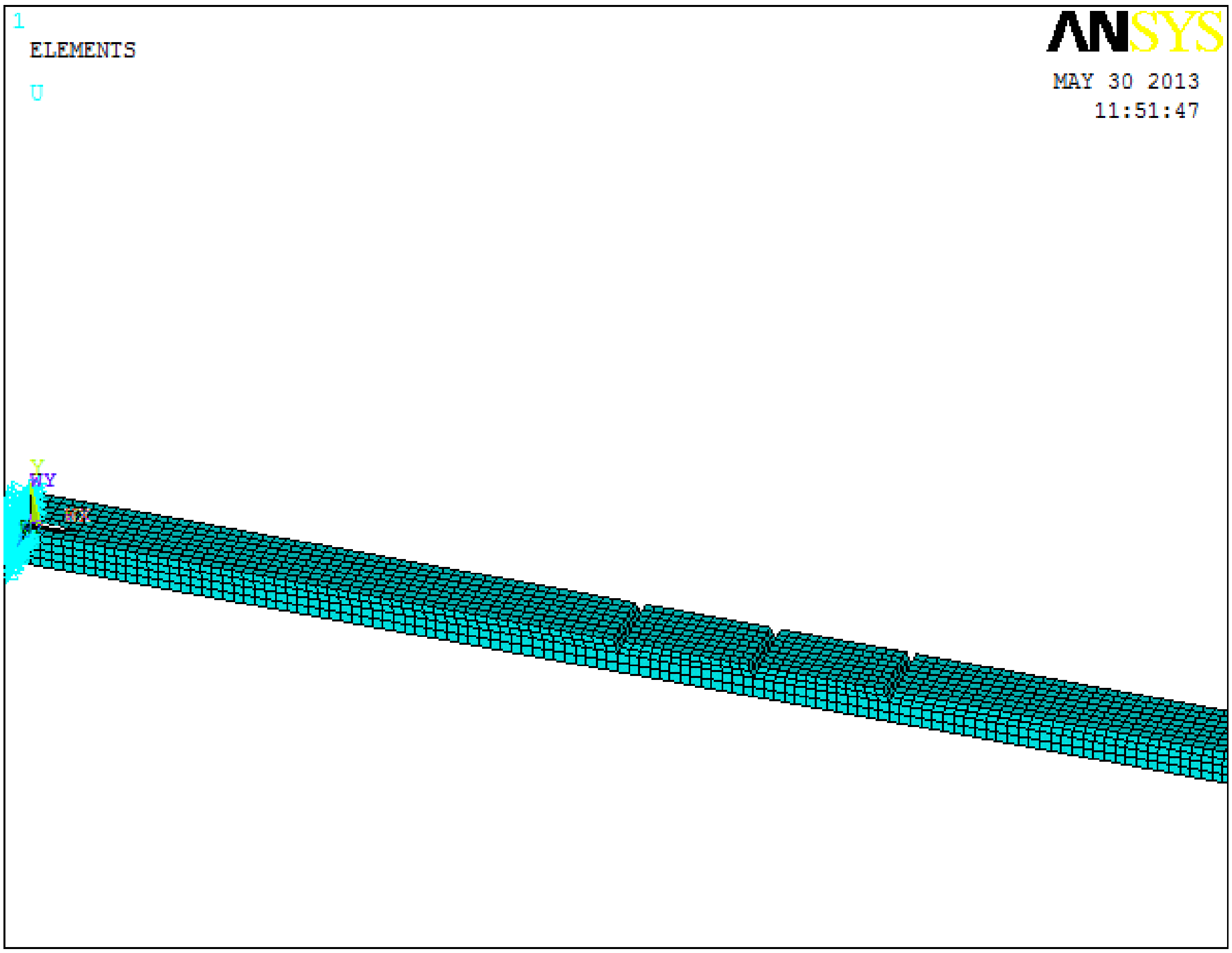

Fig. 4.18 Cantilever composite beam with multiple cracks modeled in ANSYS 13

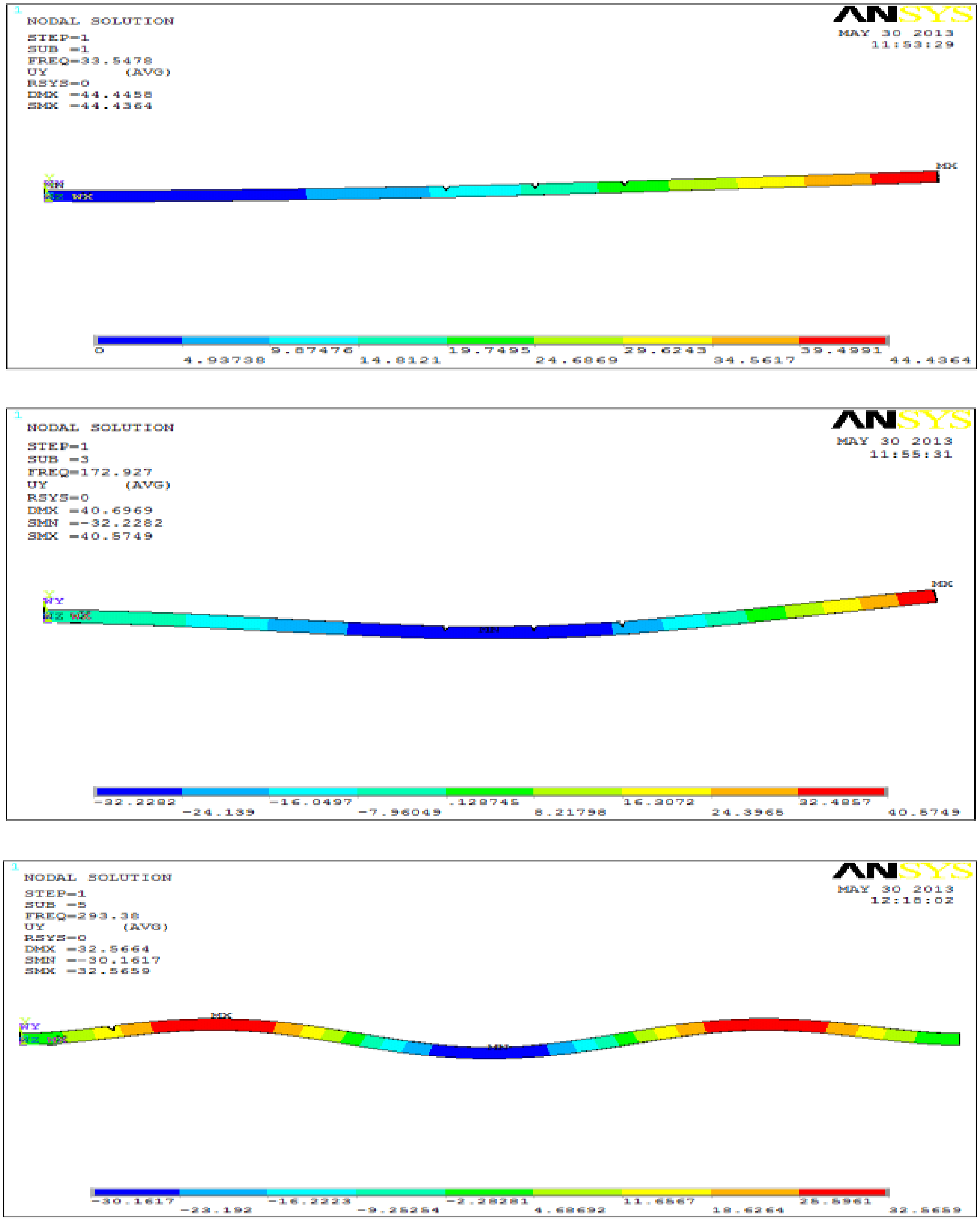

Fig. 4.19 First three mode shapes of clamped-free composite beam with multiple cracks for crack locations at L1/L= 0.45, L2/L = 0.55 and L3/L = 0.65.

Chapter 5

CONCLUSIONS

5.1 Conclusion

The following conclusions can be drawn from the present investigations of the composite beam finite element having transverse open crack i.e. v-notch. This element is versatile and can be used for static and dynamic analysis of a composite beam.

- The in-plane bending frequencies decrease, in general, as the fiber angle increases; the maximum occur at $\alpha = 0°$and decrease gradually with increasing the fiber angle up to a minimum value obtained for $\alpha =90°$.
- In case of composite beam with crack, as the angle of fibers (α) increases the value of the natural frequencies also increases. The most difference in frequency occurs when angle of fibers is zero degree.
- The non-dimensional natural frequencies is also depends upon the volume fraction of the fibers. The flexibility due to crack is high when the volume fraction of the fiber is between 0.2 and 0.8 and maximum when the fiber fractions is nearly 0.45
- Decrease in the natural frequencies become more intensive with the growth of the depth of crack.
- The increase of the beam length results in a decrease in the natural frequencies of the composite beam
- Boundary conditions have a remarkable influence on the natural frequencies. The natural frequencies for the clamped-clamped support are higher compared to clamped-free support condition.

- The first natural frequency is maximum at crack locations L1/L = 0.1 and L1/L = 0.9 and minimum at L1/L = 0.5. While the second natural frequency is minimum at crack locations L1/L = 0.3 and L1/L = 0.7.
- The effect of cracks is more pronounced near the fixed end than at far free end. It is concluded that the first, second and third natural frequencies are most affected when the cracks located at the near of the fixed end, the middle of the beam and the free end, respectively

5.2 Scope for future work

1. The vibration results obtained using ANSYS 13 can be verified by conducting experiments.
2. The dynamic stability of the composite beam with cracks
3. Static and dynamic stability of reinforced concrete beam with cracks.
4. The Vibration analysis of composite beam by introducing inclined cracks in place of transverse crack.

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